Numerical Study of Tip Vortex Cavitation

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HYDRONET 2

Multidisciplinary consortium

Simulation of sand erosion  Tip vortex Cavitation  Instability of pump-turbine  HydroPower design  Plant monitoring

To improve the Design, Manufacturing and Operation of HydroPower Plants

Experimental part

Numerical part
Tip vortex in Kaplan turbine

- Promotes cavitation.
- Flow topology of the tip-leakage flow?
- Gap width influence?
- Vortex control?
- Scale up from the model to the prototype?
OBJECTIVES

COUPLING EXPERIMENTAL AND NUMERICAL INVESTIGATION

- To investigate the gap width influence.
- To investigate the flow topology in the gap.
- To investigate the cavitation influence.

\( \text{gap=15 [mm]} \)

\( \text{gap=4 [mm]} \)
TEST CASE : NACA0009

Main characteristics

- Chord length : $c = 0.1$ m.
- Inlet velocity : $U_\infty = 10$ m/s.
- Reynolds number : $Re_c = 10^6$.
- Cross section : $1.5c \times 1.5c$.
- Gap width : $0.1c$.
- Blade incidence : $10^\circ$.
- Cavitation number : $\sigma_{expe} = 2.1$, $\sigma_{num} = 1.3$. 
Reynolds Averaged Navier-Stokes (RANS) equations

Continuity and momentum equations:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0
\]

\[
\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial (\sigma_{ij} + \tau_{ij})}{\partial x_j}
\]

With:

\[
\sigma_{ij} = \rho \nu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right);
\]

\[
\tau_{ij} = -\rho u'_i u'_j = \rho \nu_t \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]

\( \nu_t \) computed using the \( k - \omega \) SST model.
MODELLING : Cavitation

Homogeneous Relaxation Model (HRM)

Mixture density defined as:

\[ \rho = \alpha_L \rho_L + (1 - \alpha_L) \rho_V \]

Transport equation for \( \alpha_L \):

\[ \frac{\partial \alpha_L}{\partial t} + u_j \frac{\partial \alpha_L}{\partial x_j} = m_v + m_c \]

With (Kunz’s model [1]):

\[ m_v = \frac{\rho}{\rho_L} \frac{C_v}{t_\infty} \frac{\min (p - p_{vap}, 0)}{0.5 \rho_L U_\infty^2} \]

\[ m_c = \frac{\rho}{\rho_L} \frac{C_c}{t_\infty} \frac{\alpha_L^2}{\max (p - p_{vap}, 0)} \frac{\max (p - p_{vap}, 0.01 p_{vap})}{\max (p - p_{vap}, 0.01 p_{vap})} \]

NUMERICAL DOMAIN

Computational domain

- Length \( L = 8c : 2c \) upstream and \( 5c \) downstream.
- Section : \( 1.5c \times 1.5c \).

Structured mesh

- 2 millions of nodes.
- 30 nodes in the gap.

Flow direction

Gap
NUMERICAL SET UP

OpenFOAM 2.1.0 : interPhaseChangeFoam algorithm

Boundary conditions

- No slip wall.
- Inlet : normal velocity $u_\infty = 10$ m/s.
- Outlet : gradient free.
- Pressure reference sets in one cell.

Numerical Schemes

- Time scheme : second order ("backward") with $\Delta t = 10^{-5}$ s.
- Convective scheme : second order based on the Total Variation Diminishing (TVD) approach ("limitedLinear" and "vanLeer").
- Laplacian scheme : second order ("linear corrected").
- Sharp interface is preserved introducing a counter-gradient in the transport equation for $\alpha_L$. 
**VORTEX CORE POSITION**

without cavitation

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<th>x/c</th>
<th>y/c (Expe)</th>
<th>y/c (RANS)</th>
</tr>
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<tr>
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<td>0.13</td>
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<td>1.5</td>
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<table>
<thead>
<tr>
<th>x/c</th>
<th>z/c (Expe)</th>
<th>z/c (RANS)</th>
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<td>0.13</td>
</tr>
<tr>
<td>1.5</td>
<td>0.16</td>
<td>0.15</td>
</tr>
</tbody>
</table>

More detailed results in:

AXIAL VORTICITY without cavitation

Position $x/c = 1$

Experiment

RANS Computation
Dimensionless streamwise velocity component in the mid-plane

Without cavitation

With cavitation
CAVITATING TIP VORTEX
VORTEX CORE?

\[ Q = \frac{1}{2} \left( ||\Omega||^2 - ||S||^2 \right) \]
VORTICITY EQUATION

\[
\frac{\partial \vec{\omega}}{\partial t} + (\vec{u} \cdot \nabla)\vec{\omega} = (\vec{\omega} \cdot \nabla)\vec{u} - \vec{\omega} \nabla \cdot \vec{u} + \nu \nabla^2 \vec{\omega} + \nu_t \nabla^2 \vec{\omega} + \frac{1}{\rho^2} \nabla \rho \wedge \nabla p
\]

RHS

Cross plane at \( x/c = 0 \)

Without cavitation

With cavitation
CONCLUSION

- Non-cavitating tip vortex is captured accurately compared to the experiment.
- Vorticity at the vortex core is under-estimated
  - Pressure drop at the vortex core is also under-estimated.
  - Cavitating computations are performed at a lower cavitation number ($\sigma_{num} = 1.3$ instead of $\sigma_{expe} = 2.1$).
- Cavitating tip vortex shows a qualitative agreement with the experiment.
- Vortex core identification depends on the criterion chosen:
  - Maximum of the void fraction.
  - Maximum of the Q-criterion.
- The production of streamwise vorticity is negative in case of cavitation.

A detail analysis is expected in an upcoming paper.
THANK YOU FOR YOUR ATTENTION