FUNDAMENTALS OF TEXTURE PROCESSING FOR BIOMEDICAL IMAGE ANALYSIS

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Biomedical texture analysis: background
Defining texture processes
- Notations, sampling and texture functions
Texture operators, primitives and invariances
- Multiscale analysis
  - Operator scale and uncertainty principle
  - Region of interest and operator aggregation
- Multidirectional analysis
  - Isotropic versus directional operators
  - Importance of the local organization of image directions
Conclusions
References
BACKGROUND – RADIOMICS - HISTOPATHOLOGICOMICS

● **Personalized medicine** aims at enhancing the patient’s quality of life and prognosis

● Tailored treatment and medical decisions based on the molecular composition of diseased tissue

● **Current limitations** [Gerlinger2012]

● Molecular analysis of tissue composition is invasive (biopsy), slow and costly

● Cannot capture molecular heterogeneity
BACKGROUND – RADIOMICS - HISTOPATHOLOMICS

• Huge potential for computerized medical image analysis

• Explore and reveal tissue structures related to tissue composition, function, ….

• Local quantitative image feature extraction

• Supervised and unsupervised machine learning

Supervised learning, big data

quant. feat. #1

quant. feat. #2

+ malignant, nonresponder

○ malignant, responder

× benign

★ undefined

pre-malignant
**Background – Radiomics - Histopatholomics**

- Huge potential for computerized **medical image analysis**
- Create **imaging biomarkers** to predict diagnosis, prognosis, treatment response [Aerts2014]

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<tr>
<td>Reuse existing diagnostic images</td>
<td>✓ radiology data¹</td>
<td>✓ digital pathology</td>
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<tr>
<td>Capture tissue heterogeneity</td>
<td>✓ 3D neighborhoods (e.g., 512x512x512)</td>
<td>✓ large 2D regions (e.g., 15,000x15,000)</td>
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<tr>
<td>Analytic power beyond naked eyes</td>
<td>✓ complex 3D tissue morphology</td>
<td>✓ exhaustive characterization of 2D tissue structures</td>
</tr>
<tr>
<td>Non-invasive</td>
<td>✓</td>
<td>✗</td>
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¹e.g., X-ray, Ultrasound, CT, MRI, PET, OCT, …
Huge potential for computerized medical image analysis

Explore and reveal tissue structures related to tissue composition, function, ....

Local quantitative image feature extraction

Supervised and unsupervised machine learning

Specific to texture!
OUTLINE

● Biomedical texture analysis: background
● Defining texture processes
  ● Notations, sampling and texture functions
● Texture operators, primitives and invariances
  ● Multiscale analysis
    ● Operator scale and uncertainty principle
    ● Region of interest and operator aggregation
  ● Multidirectional analysis
    ● Isotropic versus directional operators
    ● Importance of the local organization of image directions
● Conclusions
● References
Definition of texture

- Everybody agrees that nobody agrees on the definition of “texture” (context-dependent)
- “coarse”, “edgy”, “directional”, “repetitive”, “random”, …
- Oxford dictionary: “the feel, appearance, or consistency of a surface or a substance”
- [Haidekker2011]: “Texture is a systematic local variation of the image values”
- [Petrou2011]: “The most important characteristic of texture is that it is scale dependent. Different types of texture are visible at different scales”
Spatial scales and directions in images are fundamental for visual texture discrimination [Blakemore1969, Romeny2011]

- Relating to directional frequencies (shown in Fourier)
**Computerized Texture Analysis**

- Spatial **scales** and **directions** in images are fundamental for visual texture discrimination [Blakemore1969, Romeny2011]

![Scales](image1.png)  ![Directions](image2.png)

- Most approaches are leveraging these two properties
  - **Explicitly**: Gray-level co-occurrence matrices (GLCM), run-length matrices (RLE), directional filterbanks and wavelets, Fourier, histograms of gradients (HOG), local binary patterns (LBP)
  - **Implicitly**: Convolutional neural networks (CNN), scattering transform, topographic independent component analysis (TICA)
NOTATIONS, SAMPLING AND TEXTURE FUNCTIONS

- **2-D continuous texture functions in space and Fourier**
  - Cartesian coordinates: $f(x), x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2$, $f(x) \xrightarrow{\mathcal{F}} \hat{f}(\omega) = \int_{\mathbb{R}^2} f(x) e^{-j(\omega,x)} \, dx$, $\omega \in \mathbb{R}^2$
  - Polar coordinates: $f(r, \theta), r \in \mathbb{R}^+, \theta \in [0, 2\pi)$, $f(r, \theta) \xrightarrow{\mathcal{F}} \hat{f}(\rho, \phi), \rho \in \mathbb{R}^+, \phi \in [0, 2\pi)$

- **2-D digital texture functions**
  - Cartesian coordinates: $f(k), k = \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} \in \mathbb{Z}^2$
  - Polar coordinates: $f(R, \Theta), R \in \mathbb{Z}^+, \Theta \in [0, 2\pi)$

- **Sampling (Cartesian)**
  - Increments in $(k_1, k_2)$ corresponds to physical displacements in $\mathbb{R}^2$ as

\[
\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \Delta x_1 \cdot k_1 \\ \Delta x_2 \cdot k_2 \end{pmatrix}
\]
NOTATIONS, SAMPLING AND TEXTURE FUNCTIONS

- **3-D continuous texture functions in space and Fourier**
  - Cartesian coordinates: \( f(x), x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \), \( f(x) \xrightarrow{\mathcal{F}} \hat{f}(\omega) = \int_{\mathbb{R}^3} f(x)e^{-j(\omega,x)}dx, \omega \in \mathbb{R}^3 \)
  - Polar coordinates: \( f(r, \theta, \phi), \ r \in \mathbb{R}^+, \theta \in [0, 2\pi), \phi \in [0, 2\pi) \)

- **3-D digital texture functions**
  - Cartesian coordinates: \( f(k), k = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} \in \mathbb{Z}^3 \)
  - Polar coordinates: \( f(R, \Theta, \Phi), \ R \in \mathbb{Z}^+, \Theta \in [0, 2\pi), \Phi \in [0, 2\pi) \)

- **Sampling (Cartesian)**
  - Increments in \((k_1, k_2, k_3)\) corresponds to physical displacements in \(\mathbb{R}^3\) as

\[
\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \Delta x_1 \cdot k_1 \\ \Delta x_2 \cdot k_2 \\ \Delta x_3 \cdot k_3 \end{pmatrix}
\]
We consider a texture function $f(x)$ as a realization of a spatial stochastic process of $\mathbb{R}^d$

$$\{X_m, m \in \mathbb{R}^{M_1 \times \cdots \times M_d}\}$$

where $X_m$ is the value at the spatial position indexed by $m$.

- The values of $X_m$ follow one or several probability density functions $f_{X_m}(q)$

**Examples**

- Moving average Gaussian
  $m \in \mathbb{R}^{128 \times 128}$

- Pointwise Poisson
  $m \in \mathbb{R}^{32 \times 32}$

- Biomedical: lung fibrosis in CT
  $m \in \mathbb{R}^{84 \times 84}$
Stationarity of spatial stochastic processes

A spatial process \( \{ X_m, m \in \mathbb{R}^{M_1 \times \cdots \times M_d} \} \) is \textit{stationary} if the probability density functions \( f_{X_m}(q) \) are equivalent for all \( m \).

Example: heteroscedastic moving average Gaussian process

\[
\begin{align*}
  f_{a, X_m}(q) &= \frac{1}{1\sqrt{2\pi}} e^{-\frac{(q-0)^2}{21^2}} \\
  f_{b, X_m}(q) &= \frac{1}{3\sqrt{2\pi}} e^{-\frac{(q-0)^2}{23^2}}
\end{align*}
\]
NOTATIONS, SAMPLING AND TEXTURE FUNCTIONS

- Stationarity of textures and human perception / tissue biology
  - Strict/weak process stationarity and texture class definition is not equivalent
    - Outex “canvas039”: stationary?
    - Brain glioblastoma in T1-MRI: stationary?

- Image analysis tasks when textures are considered as
  - Stationary (wide sense): texture classification
  - Non-stationary: texture segmentation
Biomedical texture analysis: background

Defining texture processes
- Notations, sampling and texture functions

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Texture operators

A $d$-dimensional texture analysis approach is characterized by a set of $N$ local operators centered at the position $m$:

$$g_n(x, m) : \mathbb{R}^{M_1 \times \cdots \times M_d} \rightarrow \mathbb{R}^P, \quad n = 1, \ldots, N$$

$g_n$ is local in the sense that each element $(g_n(x, m))_{p=1,\ldots,P}$ only depends on a $L_1 \times \cdots \times L_d$ subregion of $x$.

The subregion $L_1 \times \cdots \times L_d$ is the support of $g_n$.

$g_n$ can be linear (e.g., wavelets) or non-linear (e.g., median, GLCMs, LBPs).

For each position $m$, $g_n$ maps the texture function $f(x)$ into a $P$-dimensional space:

$$g_n(f(x), m) : \mathbb{R}^{M_1 \times \cdots \times M_d} \rightarrow \mathbb{R}^P$$
From texture operators to texture **measurements** (i.e., features)

- The operator $g_n(x, m)$ is typically applied to all positions $m$ of the image by “sliding” its window $L_1 \times \cdots \times L_d$ over the image.

- Regional texture measurements $\mu \in \mathbb{R}^P$ can be obtained from the aggregation of $g_n(f(x), m)$ over a region of interest $M$.

- For instance, **integration** can be used to aggregate $g_n(f(x), m)$ over $M$.

  - e.g., average:

    $$\mu = \left( \begin{array}{c} \mu_1 \\ \vdots \\ \mu_P \end{array} \right) = \frac{1}{|M|} \int_M (g_n(f(x), m))_{p=1,\ldots,P} \, dm$$
Texture primitives

A “texture primitive” (also called “texel”) is a fundamental elementary unit (i.e., a building block) of a texture class [Haralick1979, Petrou2006]

Intuitively, given a collection of texture functions $f_j$, an appropriate set of texture operators $g_n$ must be able to:

(i) Detect and quantify the presence of all distinct primitives in $f_j=1,\ldots,J$

(ii) Characterize the spatial relationships between the primitives (e.g., geometric transformations, density) when aggregated
TEXTURE OPERATORS AND PRIMITIVES

- General-purpose texture operators
  - In general, the texture primitives are neither well-defined, nor known in advance (e.g., biomedical tissue)

  ![Image](image-url)

  **2-D lung tissue in CT images** [Depeursinge2012a]

  - **healthy**
  - **emphysema**
  - **ground glass**
  - **fibrosis**
  - **micronodules**

  **3-D normal and osteoporotic bone in μCT** [Dumas2009]

- General-purpose operator sets are useful to estimate the primitives
- How to build such operator sets?


**TEXTURE OPERATORS AND PRIMITIVES**

- **General-purpose** texture operators
  - The exhaustive analysis of spatial scales and directions is computationally expensive when the support $L_1 \times \cdots \times L_d$ of the operators is large: choices are required

- Directions (e.g., GLCMs, RLE, HOG)
  
  ![Direction Diagrams](image)

- Scales
  
  2-D GLCMs with various spatial distances $d$ [Haralick1979]

  2-D isotropic dyadic wavelets in Fourier [Chenouard2011]
Invariances of the texture operators to geometric transformations can be desirable

- E.g., scaling, rotations and translations

Invariances of texture operators can be enforced

- Example with 2-D Euclidean transforms (i.e., rotation and translation)

\[ g(f(x), m) = g(f(R_{\theta_0}x - x_0), m), \quad \forall x_0 \in \mathbb{R}^2, \theta_0 \in [0, 2\pi) \]

with the rotation matrix

\[
R_{\theta_0} = \begin{pmatrix}
\cos \theta_0 & -\sin \theta_0 \\
\sin \theta_0 & \cos \theta_0
\end{pmatrix}
\]
Computer vision versus biomedical imaging

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<thead>
<tr>
<th></th>
<th>Computer vision</th>
<th>Biomedical image analysis</th>
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<tbody>
<tr>
<td>translation</td>
<td>translation-invariant</td>
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</tr>
<tr>
<td>rotation</td>
<td>rotation-invariant</td>
<td>rotation-invariant</td>
</tr>
<tr>
<td>scale</td>
<td>scale-invariant</td>
<td>multi-scale</td>
</tr>
</tbody>
</table>

[Lazebnik2005]

[Fig. 10. (a) A digitized histopathology image of Grade 4 CaP and different graph-based representations of tissue architecture via Delaunay Triangulation, (b) results at scale 1, (c) results at scale 2, and (d) results at scale 3. Note that only areas determined as suspicious at lower scales are considered for further analysis at higher scales.]

[Gurcan2009]

[Fig. 12 shows the results of a hierarchical classifier for detection of prostate cancer from digitized histopathology. Fig. 12(a) is the original image with the tumor region (ground truth) in black contour, (b) results at scale 1, (c) results at scale 2, and (d) results at scale 3. Note that only areas determined as suspicious at lower scales are considered for further analysis at higher scales.]

Invariance of Texture Operators

- Computer vision versus biomedical imaging
The latter raises two major challenges. First, \( f(x) \) and \( \mu_x \), which hinders the spatial precision of texture \( f(x) \) and to specifically preserve the action of the integrated operators becomes diffuse when unidirectional operators are not local anymore and their joint responses become only sensitive to the global amount of image gradients. When separately integrated, the responses of unidirectional image domain becomes even more destructive when unidirectional operators are applied and rotated at each point of the image to evaluate the magnitude of their responses, i.e., the presence of the corresponding texture class. We experimentally show a wealth of digital texture patterns is tightly related to the joint information between positions and the global orientation of the image. Examining the texture responses of image gradients, one can characterize the LOID in a rotation-invariant fashion. The operators are based on steerable Euclidean transforms (also called rigid), i.e., those that can characterize the LOID in a rotation-invariant manner. LBP are not able to discriminate between the two textures classes, which outperformed the reported descriptors of methods based on local binary patterns (LBP). Multiscale analysis has been leveraged in the literature to define and discriminate the LOID and orientations in images, which is discarded by operators of the above-mentioned imaging modalities yield images of the structure of the roto-translation group. Recently proposed by Oyallon, deep convolutional networks were used for the characterization of textural patterns often requires the preservation of the joint location and orientation. We enforce the preservation of the joint location and orientation in a rotation-invariant fashion. The operators are based on steerable wavelets and a combination of local binary patterns and steerable wavelets. We introduce a new method for the characterization of natural textures, which outperforms the reported descriptors and is specifically designed to be rotation-invariant. It is based on a combination of local binary patterns and steerable wavelets and a rotation-invariant version of the local binary patterns. We enforce the preservation of the joint location and orientation in a rotation-invariant fashion. The operators are based on steerable wavelets and a combination of local binary patterns and steerable wavelets.

**Conclusions**

- **References**
**Multiscale Texture Operators**

- **Inter-patient and inter-dimension scale normalization**
  - Most medical imaging protocols yield images with various sampling steps \( \Delta x_1, \Delta x_2, \Delta x_3 \)
  - **Inter-patient** scale normalization is required to ensure the correspondance of spatial frequencies
    \[
    \Delta x_1 = \Delta x_2 = 0.4\text{mm} \\
    \Delta x_1 = \Delta x_2 = 1.6\text{mm}
    \]
  - **Inter-dimension** scale normalization is required to ensure isotropic scale/directions definition

- Inter-patient scale normalization is required to ensure the correspondance of spatial frequencies

  - Inter-dimension scale normalization is required to ensure isotropic scale/directions definition

\[
\Delta x_2 \quad \Delta x_1 \quad \Delta x_3 \\
\Delta x'_2 \quad \Delta x'_1 \quad \Delta x'_3
\]
MULTISCALE TEXTURE OPERATORS

- Which scales for texture measurements?

Two aspects:

A. How to define the size(s) of the operator(s) $L_1^n \times \cdots \times L_d^n$?

B. How to define the size of the region of interest $M$?

- Increasingly small operator size $L^1 > L^2 > \cdots > L^N$

Original image $f(x)$

$\mu_M$: concatenated measurements from multiscale operators $g_{n=1,...,N}(f(x), m)$ aggregated over $M$
Which scales for texture measurements?

Two aspects:

A. How to define the size(s) of the operator(s) $L_1^n \times \cdots \times L_d^n$?

B. How to define the size of the region of interest $M$?

For instance, integration can be used to aggregate $g(x, m)$ over $M$, e.g., average: $\mu_M = \frac{1}{|M|} \sum_{m} g(f(x), m)$.

The operator is typically applied to all positions of the image by "sliding" its window over the image.

Regional texture measurements can be obtained from the aggregation of $g(x, m)$ over a region.

$\mu_M$: concatenated measurements from multiscale operators $g_{n=1,\ldots,N}(f(x), m)$ aggregated over $M$.

original image $f(x)$

increasingly small operator size $L^1 > L^2 > \cdots > L^N$
Size and spectral coverage of operators

- **Uncertainty principle**: operators cannot be well located both in space and Fourier [Petrou2006]

- In 2D, the trade-off between the spatial support (i.e., $L_1 \times L_2$) and frequency support (i.e., $\Omega_1 \times \Omega_2$) of the operators is given by

$$L_1^2 \Omega_1^2 L_2^2 \Omega_2^2 \geq \frac{1}{16}$$
MULTISCALE TEXTURE OPERATORS

- Size and spectral coverage of operators
  - Becomes a problem in the case of non-stationary texture:
    - accurate spatial localization ↔ poor spectrum characterization

Gaussian window:
\( \sigma = 3.2\text{mm} \)
Size and spectral coverage of operators

Becomes a problem in the case of non-stationary texture:

- Accurate spatial localization
- Poor spectrum characterization

Gaussian window:
\[ \sigma = 38.4 \text{mm} \]
• Size and spectral coverage of operators

• Becomes a problem in the case of non-stationary texture:

The spatial support should have the minimum size that allows rich enough texture-specific spectral characterization.

Gaussian window:
\( \sigma = 38.4 \text{mm} \)
**Multiscale Texture Operators**

- **Other consequence:**
  - Large influence of proximal objects when the support of operators is larger than the region of interest.
  - Example with band-limited operators (2D isotropic wavelets) and lung boundary [Ward2015, Depeursinge2015a]

- Tuning the shape/bandwidth was found to have a strong influence on lung tissue classification accuracy.

![Graphs showing spatial and spectral localization](image)

- Better spatial localization
- Worse spectral localization
MULTISCALE TEXTURE OPERATORS

- Which scales for texture measurements?
- Two aspects:

  A. How to define the size(s) of the operator(s) $L_1^n \times \cdots \times L_d^n$?
  B. How to define the size of the region of interest $M$?

Increasingly small operator size $L_1 \times \cdots \times L_d$.

Original image $f(x)$.

Depeursinge2012b
MULTISCALE TEXTURE OPERATORS

- How large must be the region of interest $\mathbf{M}$?
  - No more than enough to evaluate texture stationarity in terms of human perception / tissue biology
  - Example with operator: undecimated isotropic Simoncelli’s dyadic wavelets [Portilla2000] applied to all image positions $\mathbf{m} \in \mathbb{R}^{M_1 \times M_2}$

\[
\begin{align*}
\hat{g}_1(\rho) &= \begin{cases} 
\cos\left(\frac{\pi}{2} \log_2 \left(\frac{2\rho}{\pi}\right)\right), & \frac{\pi}{4} < \rho \leq \pi \\
0, & \text{otherwise}.
\end{cases} \\
\hat{g}_2(\rho) &= \begin{cases} 
\cos\left(\frac{\pi}{2} \log_2 \left(\frac{4\rho}{\pi}\right)\right), & \frac{\pi}{8} < \rho \leq \frac{\pi}{4} \\
0, & \text{otherwise}.
\end{cases} \\
g_{1,2}(f(\rho, \phi)) &= \hat{g}_{1,2}(\rho, \phi) \cdot \hat{f}(\rho, \phi)
\end{align*}
\]

- Operators’ responses are averaged over $\mathbf{M}$

The averaged responses over the entire image does not correspond to anything visually!
How large must be the region of interest $M$?

- No more than enough to evaluate texture stationarity in terms of human perception / tissue biology

Example with operator: undecimated isotropic Simoncelli’s dyadic wavelets [Portilla2000] applied to all image positions $m \in \mathbb{R}^{M_1 \times M_2}$

$$
\hat{g}_1(\rho) = \begin{cases} 
\cos \left( \frac{\pi}{2} \log_2 \left( \frac{2\rho}{\pi} \right) \right), & 0 < \rho \leq \pi \\
0, & \text{otherwise.}
\end{cases}
$$

$$
\hat{g}_2(\rho) = \begin{cases} 
\cos \left( \frac{\pi}{2} \log_2 \left( \frac{4\rho}{\pi} \right) \right), & \frac{\pi}{8} < \rho \leq \frac{\pi}{2} \\
0, & \text{otherwise.}
\end{cases}
$$

- Operators’ responses are averaged over $M$

The averaged responses over the entire image does not correspond to anything visually!

Nor biologically!
How large must be the region of interest \( M \)?

No more than enough to evaluate texture stationarity in terms of human perception / tissue biology.

Define regions that are homogeneous in terms of operators’ responses (e.g., pixelwise clustering, graph cuts [Malik2001], Pott’s model [Storath2014]).

Nor biologically!

\[
\frac{1}{|M|} \int_M |g_2(f(x), m)| \mathrm{d}m
\]

operators' responses are averaged over the entire image does not correspond to anything visually!

Integrate over a region of interest \( \mathcal{R} \)

\[
\hat{g}_1(\varphi) = \begin{cases} 
\cos \varphi / 2 & \varphi < \varphi_0, \\
0 & \text{otherwise}
\end{cases}
\]

\[
\hat{g}_2(\varphi) = \begin{cases} 
\cos \varphi / 2 & \varphi < \varphi_0, \\
0 & \text{otherwise}
\end{cases}
\]

\[
\hat{g}_1(\varphi) = \frac{1}{L_1} \hat{g}_1(\varphi), \quad \hat{g}_2(\varphi) = \frac{1}{L_2} \hat{g}_2(\varphi)
\]
MULTISCALE TEXTURE OPERATORS

- How large must be the region of interest $M$?

- Example: supervised texture segmentation with undecimated isotropic Simoncelli’s wavelets and linear support vector machines (SVM)

train class 1 (128x128)  
train class 2 (128x128)  
test image (256x256)  

feature space (training)  
decision values  
predicted labels  

$\frac{1}{|M|} \int_M |g_1(f(x), m)| dm$  
$\frac{1}{|M|} \int_M |g_2(f(x), m)| dm$  

segmentation error = 0.05127
How large must be the region of interest $M$?

- Example: supervised texture segmentation with undecimated isotropic Simoncelli’s wavelets and linear support vector machines (SVM)

$M$: circular patch with $r = 30$
• How large must be the region of interest $M$?

• Example: supervised texture segmentation with undecimated isotropic Simoncelli’s wavelets and linear support vector machines (SVM)

$M$: circular patch with $r = 8$
How large must be the region of interest $M$?

- Example: supervised texture segmentation with undecimated isotropic Simoncelli's wavelets and linear support vector machines (SVM)

$M$: circular patch with $r = 128$
• How large must be the region of interest $M$?

  • Tissue properties are not homogeneous (i.e., non-stationary) over the entire organ.

  • Importance of building tissue atlases and digital phenotypes [Depeursinge2015b]
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  - Notations, sampling and texture functions

- Defining texture processes

- Texture operators, primitives and invariances
  - Multiscale analysis
    - Operator scale and uncertainty principle
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  - Multidirectional analysis
    - Isotropic versus directional operators
    - Importance of the local organization of image directions

- Conclusions

- References
MULTIDIRECTIONAL TEXTURE OPERATORS

- Which directions for texture measurements?
  - **Isotropic** operators: insensitive to image directions
    \[ g_n(r, \Omega, m) \rightarrow g_n(r, m) \]
  - Linear: 2D Gaussian filter
  - Non-linear: e.g., median filter
  - **Directional** operators
    - Linear: 2D Gaussian derivatives (e.g., Riesz wavelets [Unser2011])
      - Non-linear: 2D GLCMs
      - Other: e.g., Fourier, circular and spherical harmonics [Unser2013, Ward2014], MR8 [Varma2005], HOG [Dalal2005], Simoncelli’s pyramid [Simoncelli1995], curvelets [Candes2000]

Other: e.g., RLE [Galloway1975]
MULTIDIRECTIONAL TEXTURE OPERATORS

- **Which directions** for texture measurements?
- Is directional information important for texture discrimination?

\[ f(\mathbf{x}) \xrightarrow{\mathcal{F}} \hat{f}(\omega) \]

\[ f(\mathbf{x}) \xrightarrow{\mathcal{F}} \hat{f}(\omega) \]
MULTIDIRECTIONAL TEXTURE OPERATORS

- Which directions for texture measurements?
- Importance of the local organization of image directions (LOID)
- i.e., how directional structures intersect
MULTIDIRECTIONAL TEXTURE OPERATORS

- Which directions for texture measurements?
  - Isotropic and unidirectional operators can hardly characterize the LOIDs, especially when aggregated over a region $\mathbf{M}$ [Sifre2014, Depeursinge2014b]

- Example of feature representation when integrated over $\mathbf{M} \equiv$ entire image

isotropic Simoncelli wavelets

gradients along $x_1$ and $x_2$

GLCMs

$$ GLCM \text{ contrast} \quad d = 1 \ (\Delta_{k_1} = 1, \Delta_{k_2} = 0) $$
MULTIDIRECTIONAL TEXTURE OPERATORS

- Which directions for texture measurements?
  - Isotropic and unidirectional operators can hardly characterize the LOIDs, especially when aggregated over a region $M$ [Sifre2014, Depeursinge2014b]

Very poor discrimination!

Solutions proposed in a few slides…
Multidirectional Texture Operators

- **Locally rotation-invariant** operators over $L_1 \times L_2$

\[
g(f(x), m) = g(f(R_{\theta_0} x), m), \quad \forall \theta_0 \in [0, 2\pi), \quad \forall x \in \mathbb{R}^{L_1 \times L_2}
\]

- **Isotropic operators:**
  - By definition

- **Directional:**
  - **Averaging** operators' responses over all directions:

\[
\bar{\mu}_1 \text{ (e.g., GLCM contrast)}
\]

No characterization of image directions!
MULTIDIRECTIONAL TEXTURE OPERATORS

- **Locally rotation-invariant** operators over $L_1 \times L_2$

  $$g(f(x), m) = g(f(R_{\theta_0} x), m), \quad \forall \theta_0 \in [0, 2\pi), \quad \forall x \in \mathbb{R}^{L_1 \times L_2}$$

- **Locally “aligning”** directional operators
  - MR8 filterbank [Varma2005]
  - Steerable Riesz wavelets [Unser2013, Depeursinge2014b]
MULTIDIRECTIONAL TEXTURE OPERATORS

- Locally rotation-invariant operators over $L_1 \times L_2$
  
  $$g(f(x), m) = g(f(R_{\theta_0} x), m), \quad \forall \theta_0 \in [0, 2\pi), \quad \forall x \in \mathbb{R}^{L_1 \times L_2}$$

- Locally “aligning” directional operators
  
  - Maximum response 8 (MR8) filterbank [Varma2005]

- Filter responses are obtained for each pixel from the convolution of the filter and the image

- For each position $m$, only the maximum responses among gradient and Laplacian filters are kept
MULTIDIRECTIONAL TEXTURE OPERATORS

- Locally rotation-invariant operators over \( L_1 \times L_2 \)
  \[
g(f(x), m) = g(f(R_{\theta_0}x), m), \quad \forall \theta_0 \in [0, 2\pi), \quad \forall x \in \mathbb{R}^{L_1 \times L_2}
  \]

- Locally “aligning” directional operators
  - Maximum response 8 (MR8) filterbank [Varma2005]

Yields approximate local rotation invariance

Poor characterization of the LOIDs

- Filter responses are obtained for each pixel from the convolution of the filter and the image
- For each position \( m \), only the maximum responses among gradient and Laplacian filters are kept
**MULTIDIRECTIONAL TEXTURE OPERATORS**

- **Locally rotation-invariant** operators over $L_1 \times L_2$

  
  \[ g(f(x), m) = g(f(R_{\theta_0} x), m), \quad \forall \theta_0 \in [0, 2\pi), \quad \forall x \in \mathbb{R}^{L_1 \times L_2} \]

- **Locally “aligning” directional operators**

  - **Local binary patterns (LBP)** [Ojala2002]
    1) define a **circular neighborhood**
    2) Binarize and build a number that encode the LOIDs
    
    \[ U_8(1, 0) = 10101010 = 170 \]

  - **Rotation-invariant LBP** [Ahonen2009]

    \[ H_8(1, u) = \sum_{r=0}^{7} h_I(U_8(1, r)) e^{-j2\pi ur/8} \]

    The new measures $\mu_p = |H_8(1, u)|$ are independent of the rotation $r$. 

  4) make codes invariant to circular shifts
MULTIDIRECTIONAL TEXTURE OPERATORS

- Locally rotation-invariant operators over $L_1 \times L_2$
  \[ g(f(x), m) = g(f(R_{\theta_0}x), m), \quad \forall \theta_0 \in [0, 2\pi), \quad \forall x \in \mathbb{R}^{L_1 \times L_2} \]

- Locally “aligning” directional operators

  \[ \text{Encodes the LOIDs independently from their local orientations!} \]

  \[ \text{Requires binarization…} \]

  \[ \text{Spherical sequences are undefined in 3D…} \]

  \[ g(x, m) = g(f(R_{\theta}x), m) \]

  \[ \mu_p = |H_8(1, u)| \text{ are independent of the rotation } r \]
MULTIDIRECTIONAL TEXTURE OPERATORS

- **Locally rotation-invariant** operators over $L_1 \times L_2$
  
  $$g(f(x), m) = g(f(R_{\theta_0}x), m), \quad \forall \theta_0 \in [0, 2\pi), \quad \forall x \in \mathbb{R}^{L_1 \times L_2}$$

- **Locally “aligning” directional** operators
  
  - Steerable Riesz wavelets [Unser2013, Depeursinge2014b]
  
  - Operators: $N$th-order multi-scale image derivatives

  $$N = 1$$

  ![Images](#)
MULTIDIRECTIONAL TEXTURE OPERATORS

- Locally rotation-invariant operators over $L_1 \times L_2$

$$g(f(x), m) = g(f(R_{\theta_0}x), m), \quad \forall \theta_0 \in [0, 2\pi), \quad \forall x \in \mathbb{R}^{L_1 \times L_2}$$

- Locally “aligning” directional operators

  - Steerable Riesz wavelets [Unser2013, Depeursinge2014b]

    - Operators: $N$th-order multi-scale image derivatives

\[ N = 1 \]

$$g_{(1,0)}(x, m) \quad g_{(0,1)}(x, m) \quad g_{(2,0)}(x, m) \quad g_{(1,1)}(x, m) \quad g_{(0,2)}(x, m)$$

\[ N = 2 \]

$$g_{(3,0)}(x, m) \quad g_{(2,1)}(x, m) \quad g_{(1,2)}(x, m) \quad g_{(0,3)}(x, m)$$

\[ N = 3 \]
 MULTIDIRECTIONAL TEXTURE OPERATORS

- Locally rotation-invariant operators over $L_1 \times L_2$

$$g(f(x), m) = g(f(R_{\theta_0}x), m), \; \forall \theta_0 \in [0, 2\pi), \; \forall x \in \mathbb{R}^{L_1 \times L_2}$$

- Locally “aligning” directional operators
  - Steerable Riesz wavelets [Unser2013, Depeursinge2014b]
    - Steerability:

$$g_{(1,0)}(R_{\theta_0}x, 0) = \cos \theta_0 \; g_{(1,0)}(x, 0) + \sin \theta_0 \; g_{(0,1)}(x, 0)$$

\[ = -0.878 \quad + \quad -0.479 \]
MULTIDIRECTIONAL TEXTURE OPERATORS

- Locally rotation-invariant operators over $L_1 \times L_2$

\[ g(f(x), m) = g(f(R_{\theta_0} x), m), \quad \forall \theta_0 \in [0, 2\pi), \quad \forall x \in \mathbb{R}^{L_1 \times L_2} \]

- Locally “aligning” directional operators
  - Steerable Riesz wavelets [Unser2013, Depeursinge2014b]
    - Local rotation-invariance:
      \[
      \theta_{\text{max}}(m) := \arg \max_{\theta_0 \in [0, 2\pi)} \left( \cos \theta_0 g_{(1,0)}(f(x), m) + \sin \theta_0 g_{(0,1)}(f(x), m) \right)
      \]

\[
\Rightarrow \mu_M = \frac{1}{|M|} \int_M \left( g_{(1,0)}(f(R_{\theta_{\text{max}}(m)} x), m) \right)^2 \, dm
\]
MULTIDIRECTIONAL TEXTURE OPERATORS

- **Locally rotation-invariant** operators over $L_1 \times L_2$

  $$g(f(x), m) = g(f(R_{\theta_0} x), m), \quad \forall \theta_0 \in [0, 2\pi), \quad \forall x \in \mathbb{R}^{L_1 \times L_2}$$

- **Locally “aligning” directional operators**

  - Encodes the LOIDs independently from their local orientations!
  - No binarization required!
  - Available in 3D [Chenouard2012, Depeursinge2015a], and combined with feature learning [Depeursinge2014b].
MULTIDIRECTIONAL TEXTURE OPERATORS

- Operators characterizing the LOIDs
MULTIDIRECTIONAL TEXTURE OPERATORS

- Operators characterizing the LOIDs

GLCMs

Riesz wavelets ($N = 2$)

aligned Riesz wavelets ($N = 2$)
• Isotropic or directional analysis? [Depeursinge2014b]

• Outex [Ojala2002]: 24 classes, 180 images/class, 9 rotation angles in \([0^\circ; 90^\circ]\)

• Texture classification: linear SVMs trained with unrotated images only

---

![Image of texture images and graph](image-url)
MULTIDIRECTIONAL TEXTURE OPERATORS

- Isotropic or directional analysis? [Depeursinge2014b]

- Outex [Ojala2002]: 24 classes, 180 images/class, 9 rotation angles in $[0°; 90°]$

Isotropic operators (i.e., $N = 0$) perform best when not aligned!

- Texture classification: linear SVMs trained with unrotated images only
Biomedical texture analysis: background

Defining texture processes
  • Notations, sampling and texture functions

Texture operators, primitives and invariances
  • Multiscale analysis
    • Operator scale and uncertainty principle
    • Region of interest and operator aggregation
  • Multidirectional analysis
    • Isotropic versus directional operators
    • Importance of the local organization of image directions

Conclusions

References
We presented a general framework to describe and analyse texture information in 2D and 3D.

Tissue structures in 2D/3D medical images contain extremely rich and valuable information to optimize personalized medicine in a non-invasive way.

Invisible to the naked eye!
CONCLUSIONS

- Biomedical textures are realizations of complex non-stationary spatial stochastic processes.
- General-purpose image operators are necessary to identify data-specific discriminative scales and directions.

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<thead>
<tr>
<th>Scales</th>
<th>Texture Operators</th>
<th>Region of Interest and Aggregation</th>
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<td></td>
<td>uncertainty principle</td>
<td>averaging operators’ responses</td>
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<td>f(x)</td>
<td>( \hat{f}(\omega) )</td>
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<th>Directions</th>
<th>Isotropic versus Directional</th>
<th>Importance of LOIDs</th>
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<tr>
<td>classification accuracy</td>
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John-Paul Ward, UCF

Source code and data available!
https://sites.google.com/site/btamiccai2015/adrien.depeursinge@epfl.ch
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