Basin Stability for Quantifying Stability of Power Grids

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• Stability concepts
• Basin stability for complex networks
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Nonlinear Sciences

Start in 1665 by Christiaan Huygens:

Discovery of phase synchronization, called sympathy
Pendulum Clocks

• Christiaan Huygens:

Pendulum clocks hanging at the same wooden beam (half-timber house)

It is quite worth noting that when we suspended two clocks so constructed from two hooks imbedded in the same wooden beam, the motions of each pendulum in opposite swings were so much in agreement that they never receded the last bit from each other...Further, if this agreement was disturbed by some interference, it reestablished itself in a short time...after a careful examination I finally found that the cause of this is due to the motion of the beam, even though this is hardly perceptible (Huygens, 1673)
Basic feature of a complex system

ability for **self-organization** non-trivial structure formation (some times unexpected)

here synchronization due to almost hidden coupling
Types of Synchronization in Complex Processes

- **phase synchronization**
  
  phase difference bounded, a zero Lyapunov exponent becomes negative (phase-coherent)
  
  (Rosenblum, Pikovsky, Kurths, 1996)

- **generalized synchronization**
  
  a positive Lyapunov exponent becomes negative, amplitudes and phases interrelated
  
  (Rulkov, Sushchik, Tsimring, Abarbanel, 1995)

- **complete synchronization**
  
  (Fujisaka, Yamada 1983)
Complex Networks

Evolving Networks
Network of Networks
Interconnected Networks
Interdependent Networks
Multiplex Networks
Multilayer Networks…
Power Grids

Intended Solution:

stable synchronized behaviour along the whole network (of networks)
November 9, 1965

Large Blackout in Northeast of US

> 30 Mio people up to 12 hours without electricity
(Highly probable) Cause:

service operators installed a wrong protective relais in a power station near the Niagara Falls…

start 17:16 – cascade effect during 4 minutes monster blackout
Stability
Alexandr Mikhailovich Lyapunov

• Lyapunov was the first to consider modifications necessary in *nonlinear systems* to the linear theory of stability based on linearizing near a point of equilibrium.

• The equilibrium $x_\varepsilon$ of the system is said to be *Lyapunov stable*, if for every ($\forall \varepsilon > 0$) and ($\forall t_0$), there exists a $\delta = \delta(t_0, \varepsilon) > 0$ such that, if $|x(t_0) - x_\varepsilon| < \delta$, then $|x(t) - x_\varepsilon| < \varepsilon$, for every $t \geq 0$.

• Extension to asymptotical and exponential stability.
Stability of Complex Networks
(Synchronized Dynamics)
Weighted Network of $N$ Identical Oscillators

\[
\dot{x}_i = F(x_i) + \sigma \sum_{j=1}^{N} W_{ij} A_{ij} [H(x_j) - H(x_i)],
\]

\[
= F(x_i) - \sigma \sum_{j=1}^{N} G_{ij} H(x_j), \quad i = 1, \ldots, N,
\]

$F$ – dynamics of each oscillator

$H$ – output function

$G$ – coupling matrix combining adjacency $A$ and weight $W$

$G_{ij} = -W_{ij}$ for $i \neq j$

$G_{ii} = \sum_j W_{ij} A_{ij} = S_i$

$S_i$ – intensity of node $i$ (includes topology and weights)
General Condition for Synchronizability

Stability of synchronized state

$$\{x_i = s, \forall i \mid \dot{s} = F(s)\}$$

N eigenmodes of

$$\dot{\xi}_i = [DF(s) - \sigma \lambda_i DH(s)]\xi_i,$$

$$\lambda_i \text{ \ ith eigenvalue of } G$$
Main results

Synchronizability universally determined by:

- mean degree $K$ and

- heterogeneity of the intensities

$$\frac{S_{\text{max}}}{S_{\text{min}}} \quad \text{or} \quad \frac{\Omega}{S_{\text{min}}}$$

- minimum/maximum intensities

(Motter, Zhou, Kurths, PRL 2006)
Stability of Networks

Synchronizability –
Master Stability Formalism

Pecora&Carroll (1998) –

based on local stability
Synchronizability – Master Stability Formalism
(Pecora&Carrol (1998))

Synchronizability Ratio

\[ R = \frac{\lambda_{\text{max}}}{\lambda_{\text{min}}} \]

Stability Interval for coupling strength \( K \)

\[ K \in I_s = \left( \frac{\alpha_1}{\lambda_{\text{min}}}, \frac{\alpha_2}{\lambda_{\text{max}}} \right) \]

Synchronizability condition

\[ R < \frac{\alpha_2}{\alpha_1} \]
Stability/synchronizability in small-world (SW) networks

Small-world (SW) networks
(Watts, Strogatz, 1998 – WS-networks)

F. Karinthy hungarian writer –
SW hypothesis (1929)
Small-world Networks

- k nearest neighbour connections
- Nearest neighbour and a few long-range connections

Regular ➞ Complex Topology
\[
\dot{r}_i = F(r_i) + K \sum_j A_{ij}[H(r_j) - H(r_i)] = F(r_i) - K \sum_j L_{ij} H(r_j),
\]

\[
\begin{align*}
\dot{x}_i &= -y_i - z_i - K \sum_{i=1}^{N} L_{ij} x_j \\
\dot{y}_i &= x_i + ay_i \\
\dot{z}_i &= b + z_i(x_i - c)
\end{align*}
\]

Chosen: \( a = b = 0.2, \ c = 7.0 \ \Rightarrow \ \text{R < 37.88} \)

Chaotic Rössler oscillators, \( N = 100 \)
Main Result: SW-Network **best synchronizable** for most random SW-networks

Puzzle!
MSF – local stability
(Lyapunov stability)

How to go beyond (not only small perturbations)?

Lyapunov Functions?
Network’s Basin Stability

basin volume of a state (regime) measures likelihood of return to this state (regime)
sample-based method

Nature Physics 9, 89 (2013)
Figure 1 Thought experiment: marble on a marble track. The track is immersed in a highly viscous fluid to make the system's state space one-dimensional. Dashed arrows indicate where the marble would roll from each position. A, B and C label fixed points. Only B is stable. The green bar indicates B’s basin of attraction B. If the marble is perturbed from B to a state within the basin, it will return to B. Such perturbations are permissible. Perturbations to states outside the basin are impermissible. The dashed parabola shows the local curvature around B, fitting the true marble track poorly in most of the basin.
Network´s Basin Stability

basin volume of a state (regime) measures the likelihood of

- arrival at this state (regime)
  quantifies its relevance (M. Girvan, 2006)
- return to this state after a random perturbation
  quantifies its stability
  (Menck, Heitzig, Marwan, Kurths: Nat. Phys., 2013)
Normalized Network’s Basin Stability

- Synchronous state’s basin of attraction

\[ \mathcal{B} = \{ x \in S \mid \Phi_t(x) \to \mathcal{I} \} \]

- Subset of state space $S$ covering the system’s (weak) attractor

\[ S_{\mathcal{B} \cap Q} = \frac{\text{Vol}(\mathcal{B} \cap Q)}{\text{Vol}(Q)} \in [0, 1] \]

Normalized Basin Stability
Bernoulli-like experiment

- T experiments (different initial conditions – randomly distributed)
- M states converge to I
- Estimate M / T ➔ standard error
  \[ e := \frac{\sqrt{S_B(1 - S_B)}}{\sqrt{T}} \]
- T=500 ➔ error < 0.023
Basin Stability for the Rössler System

\[ Q := q^{**N} \quad \text{with} \quad q = [-15, 15] \times [-15, 15] \times [-5, 35] \]
Two-dimensional details of the Rössler attractor’s basin. a shows the $xy$-detail. A point $(x, y)$ refers to the initial state $(x, y, 0)$. b shows the $xz$-detail. A point $(x, z)$ refers to the initial state $(x, 0, z)$. In both panels the white region indicates the Rössler attractor’s basin of attraction. The green shape depicts a two-dimensional projection of the Rössler attractor.
Supplementary Figure S1: **Basin Stability in Rössler networks.** Expected basin stability $\langle \tilde{S} \rangle$ versus $p$. The grey shade indicates $\pm$ one standard deviation. The dashed line shows an exponential fitted to the ensemble results for $p \geq 0.15$. Solid lines are guides to the eye. **a:** $N = 100$, **b:** $N = 200$.
Synchronizability and basin stability in Watts-Strogatz (WS) networks of chaotic oscillators.

a: Expected synchronizability $R$ versus the WS model's parameter $p$. The scale of the $y$-axis was reversed to indicate improvement upon increase in $p$.

b: Expected basin stability $S$ versus $p$. The grey shade indicates one standard deviation. The dashed line shows an exponential fitted to the ensemble results for $p > 0.15$. Solid lines are guides to the eye. The plots shown were obtained for $N = 100$ oscillators of Roessler type, each having on average $k = 8$ neighbours. Choices of larger $N$ and different $k$ produce results that are qualitatively the same.
Extension to delay-coupled systems

\[
\dot{X}_i(t) = F[X_i(t)] - \sigma \sum_{j=1}^{N} g_{ij} h[X_j(t - \tau)]
\]

SW network, N = 100, chaotic Roessler oscillators, 6 neighbours each (in average) $\tau = 0.4$. 
Analysis of Power Grids

Single generator´s dynamics

\[
\begin{align*}
\dot{\theta} &= \omega \\
\dot{\omega} &= -\alpha \omega + P - K \sin(\theta - \theta_{\text{grid}}) \\
&= P_{\text{trans}}
\end{align*}
\]

\(\theta\) and \(\omega\) - phase and angular frequency of the generator´s voltage vector in a reference frame that co-rotates with the grid´s rated frequency

\(\Rightarrow\) \(\omega = 0\) - synchrony

\(P\) - net power input

- \(\alpha \omega\) - damping

\(P_{\text{trans}}\) - power flow to the grid across the transmission line
Fig. 1. (color online). Parameter space and state space of the one-machine infinite bus system. In the left panel (a), red indicates the area of stable fixed point. In the yellow area, the oscillator either converges to stable fixed point or rotates periodically depending crucially on initial values of $\theta$ and $\dot{\theta}$. White area indicates the existence of stable limit cycle. (b): Basin of attraction of the stable fixed point $\theta_f$ is indicated in the green area with $\alpha = 0.1$, $P = 0.5$ and $K = 1$. The stable fixed point and saddle are also plotted in red. The saddle is at the right side of the stable fixed point.
Fig. 2. (color online). Basin stability over parameter space of the one-machine infinite bus system. The different areas same as shown the figure 1(a) are separated by the two lines: $P/K = 1$ and the homoclinic bifurcation line. When $\alpha \to 0$, the bifurcation line is tangent to the line $\alpha = P\pi/(4\sqrt{K})$. For fixed coupling strength $K = 1$, basin stability $S$ is calculated over the parameter space by varying the value of $\alpha$ and $P$ from 0 to 1 separately.
Power Grid Model

\[ \dot{\theta}_i = \omega_i \]
\[ \dot{\omega}_i = -\alpha_i \omega_i + P_i - \sum_{j=1}^{N} K_{ij} \sin(\theta_i - \theta_j) \]

\( \theta_i \) and \( \omega_i \) denote phase and frequency of the generator at node \( i \).

Node i net generator if \( P_i > 0 \)

Node i net consumer if \( P_i < 0 \)

- damping constant
- net power input
Main Question: How stable is the synchronized regime?

\[ \omega_i = 0, \quad \dot{\omega}_i = 0 \]

Stability even in case of large perturbations at one node

Concept of basin stability

Nature Commun. 5, 3969 (2014)
Simplifications

\[ \alpha_i = \alpha \]

\[ K_{ij} = K_{ji} = K \text{ if nodes } i \text{ and } j \text{ are connected} \]

\[ K_{ij} = 0 \text{ otherwise.} \]

\[ P_i = +P \]

Net generator

\[ P_i = -P \]

Net consumer
Single-Node Basin Stability

Probability that the grid will return to its synchronous state after node $i$ has been hit by a random large perturbation

Single node basin stability

$$S_i \in [0, 1]$$

$$S_i := \frac{Vol(B_i \cap Q)}{Vol(Q)} \in [0, 1]$$

$$B_i := \{(\theta_i, \omega_i) | (\theta_j, \omega_j)_{j=1,...,N} \in B \text{ with } \theta_j = \theta_j^s \text{ and } \omega_j = 0 \text{ for all } j \neq i\}$$
Perturbed Initial Conditions only at Node $i$
Simulations

N = 100

E = 135 transmission lines

\[ \langle d \rangle = 2.7 \] \quad \text{Average Degree}

P = 1, K = 8, \quad \alpha = 0.1

1000 randomly generated networks
Figure 4: Northern European Power Grid. The grid has \( N = 236 \) nodes and \( E = 320 \) transmission lines. The load scenario was chosen randomly, with squares (circles) depicting \( N/2 \) net consumers with \( P_i = -P \) (net generators with \( P_i = +P \)). The colour scale indicates how large a node’s basin stability \( S_i \) is. Insets I-III show re-computed basin stability values after 27 lines have been added in order to ‘heal’ dead trees (see Methods). New lines are coloured red. Our simulation parameters, \( \alpha = 0.1, P = 1, \) and \( K = 8 \), imply the simplifying assumptions that all generators in the grid are of the same making and that all transmission lines are of the same voltage and impedance. These assumptions enable us to focus on the effects of the (unweighted) topology. For details, see Methods.
Stability of the Scandinavian Power Grid
Stability of the Scandinavian Power Grid
First Conclusions

• Concept of basin stability enables important new insights and principles for the design of (Smart) Power Grids

• Dead ends and dead trees strongly diminish stability ➔ they have to be avoided

• „Healing“ dead ends by addition of a few transmission lines enhances substantially stability

• For the Scandinavian power grid: addition of 27 lines (8% of the total) suffice to substantially improve stability – rather low-cost solution)
Other Approaches

- Basin stability refers to asymptotic behaviour and requires multistability.

- In many applications (power grids, brain...), transient behaviour is more important.

- Apply the concept of survivability.

→ Basin of Survival

Desirable region

Survivability $S(t)$:
Fraction of trajectories starting at $X^+$ and staying within $X^+$ the whole time $[0, t]$

t-time basin of survival $X_t^S$

$$S(t) = \frac{\text{Vol}(X_t^S)}{\text{Vol}(X^+)}$$
Example:
two-node power grid
The central green area is the infinite-time basin of survival, while the yellow and red areas contain finite-time surviving states. The union of the blue, yellow and green regions is the fixed point’s basin of attraction, while trajectories starting in the white or red regions approach the limit cycle solution. The frequency threshold is set to $\omega_{crit.} = \pm 5$. 
Stability threshold

Stability threshold is the minimal perturbation kicking the system out of the attraction basin:

$$\sigma = \inf \{ \text{dist}(x, y) | x \in A, y \in \delta B \}$$
Stability threshold vs. basin stability

$V \sim (q - \sigma)^{\frac{N+1}{2}}$

Perturbation class: magnitude $< q$

decay rate depends on the system dimension
Global minimum vs. local minimums

Stability threshold corresponds to the global minimum:

\[ \sigma = \min \{\sigma_1, \sigma_2, \sigma_3\} \]

Variation of parameters: tracing the minima
Limits of BS
Systems with Fractal Basin Boundaries

(McDonald et al. (1985), Nusse & Yorke (1996))
Basic Model: damped pendulum with time dependent forcing (Kennedy & Yorke (1991))

\[ \begin{align*}
\dot{\phi} &= \omega \\
\dot{\omega} &= -\alpha \omega + T \cos t - K \sin \phi \\
\dot{t} &= 1 \\
\end{align*} \]

pendulum’s angle \( \phi \) and frequency \( \omega \)

\[ \alpha = 0.1, \; K = 1 \text{ and } T = 7/4, \]

Wada property
Wada Property

3 subsets of state space **Wada**: any point on the boundary of one set is also on the boundary of the two others
Precision $p = 7$ – float variable

16 – full double
Stable results for basin stability (p variation)

1000 random initial conditions

$S_B$ of the four attractors at different levels of $\epsilon$. The basin stability of the black/red/orange/yellow attractor is shown by the height of the black/red/orange/yellow bar. The grey shadows indicate $\pm$ one standard deviation around the respective value of $S_B(16)$, the estimation with the highest numerical precision.
Riddled Basin Boundaries

Alexander, Yorke, You, Kann, 1992

Grebogi, Lai, Yorke, Venkataratamani, 1996

and several others later
Results NOT always stable for varying p (up to 50% variation)

limits of the method
Outlook

• Approach in its infancy – more open than solved questions
• Other sample-based approaches as survivability, threshold stability…
• Collaboration with engineers
• Prove the techniques - mathematical foundation
• To include network of network approach: time-varying price feedback, other infrastructure…
• Various other aspects in power grid dynamics – project CONDYNET (MPI Göttingen, FZ Jülich, FIAS Frankfurt, Jacobs Uni Bremen, PIK)
• Further potential applications – neuronal networks, systems biology, engineering…
Our Papers

- Nature Physics 9, 89 (2013)
- Nature Communication 5, 3969 (2014)
- EPJ ST 223, 2593 (2014)
- Phys. Rev. E 90, 022812 (2014)
- Scient. Rep. 6, 21449 (2016)
- Scient. Rep. 6, 29654 (2016)
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