

# Nonlinear Dynamics of Aggregate Load Models

**Ian A. Hiskens**

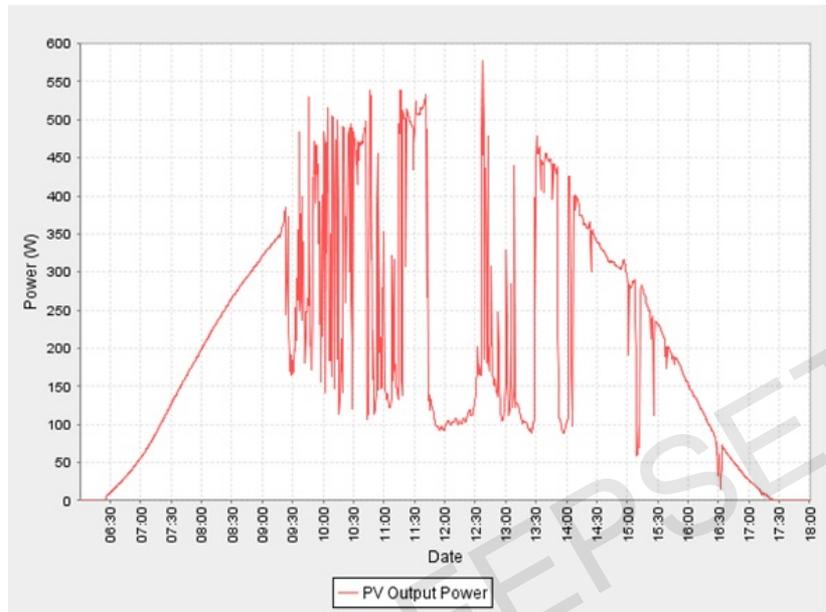
Vennema Professor of Engineering  
Professor, Electrical Engineering and Computer Science  
University of Michigan, Ann Arbor

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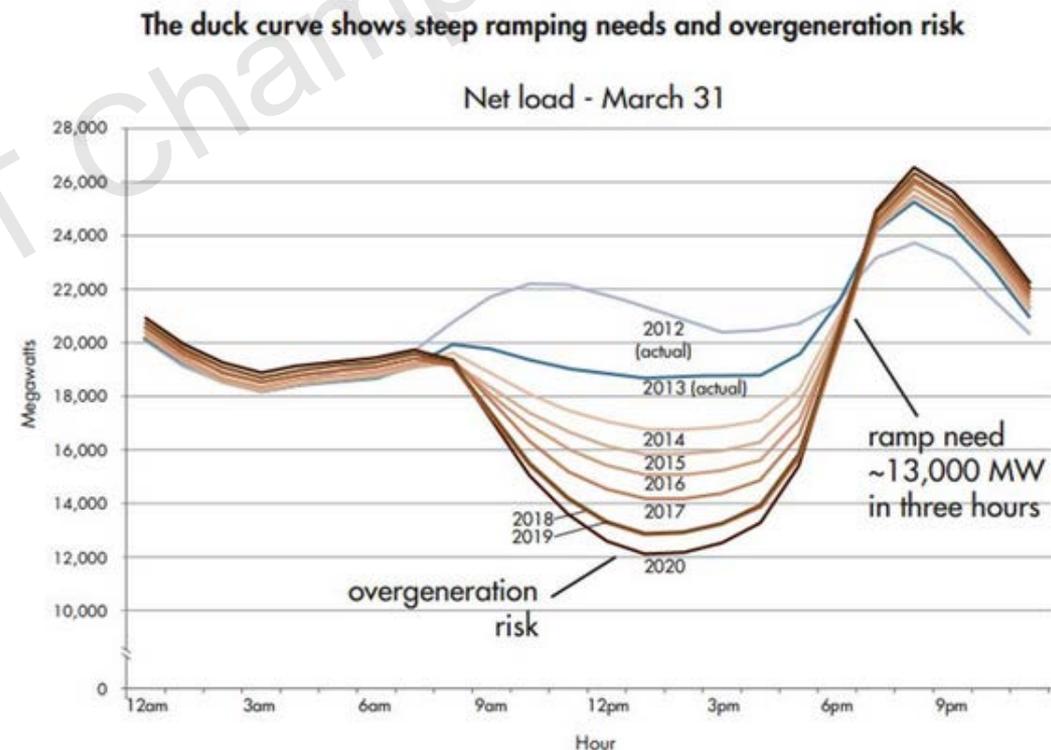
# Motivation

- Growth in renewable generation makes balancing electricity production and consumption more difficult.



Local variability

## System-wide impact



# Forms of non-disruptive load control

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- Non-disruptive load control: energy users are oblivious to the control actions.
- Large individual loads.
  - Building HVAC control.
- Large numbers of small devices.
  - Thermostatically controlled loads (TCLs).
    - Air-conditioning, refrigeration, heat pumps.
    - Offer regulation capability and/or levelize renewable generation production.
  - Electric vehicle (EV) charging.
    - Prevent undesirable loading patterns.
    - Offer regulation capability for enhancing system operation.



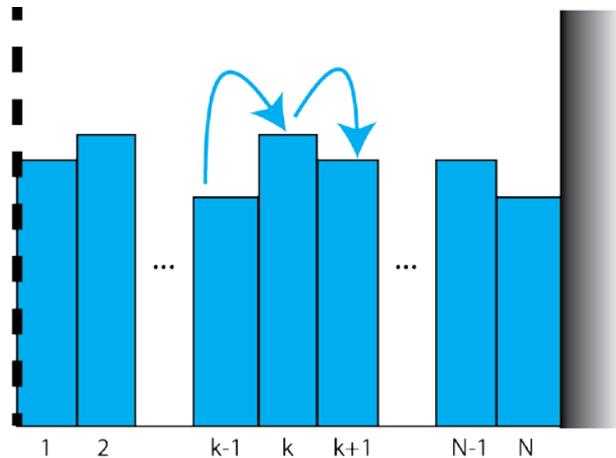
# Ensembles with natural dynamics

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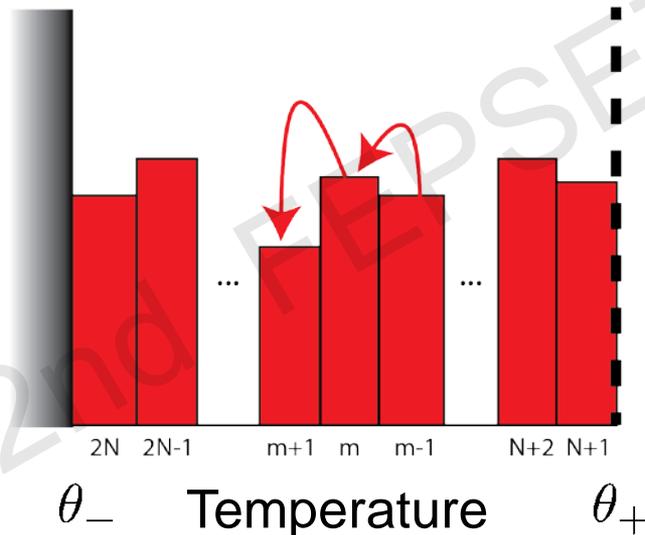
- The natural (hysteresis-based) dynamics of devices such as TCLs make regulation more challenging.
- A starting point is the development of a simplified model describing aggregate dynamic behaviour.
- The temperature associated with each TCL is influenced by random perturbations, e.g. opening doors/windows.
  - Modelled as noise.
- Every TCL has slightly different characteristics, e.g. thermal capacitance/resistance.
  - The population is heterogeneous.



# Bin model approximation



- Regions (for cooling loads):
  - Blue loads are in the off state.
  - Red loads are in the on state.



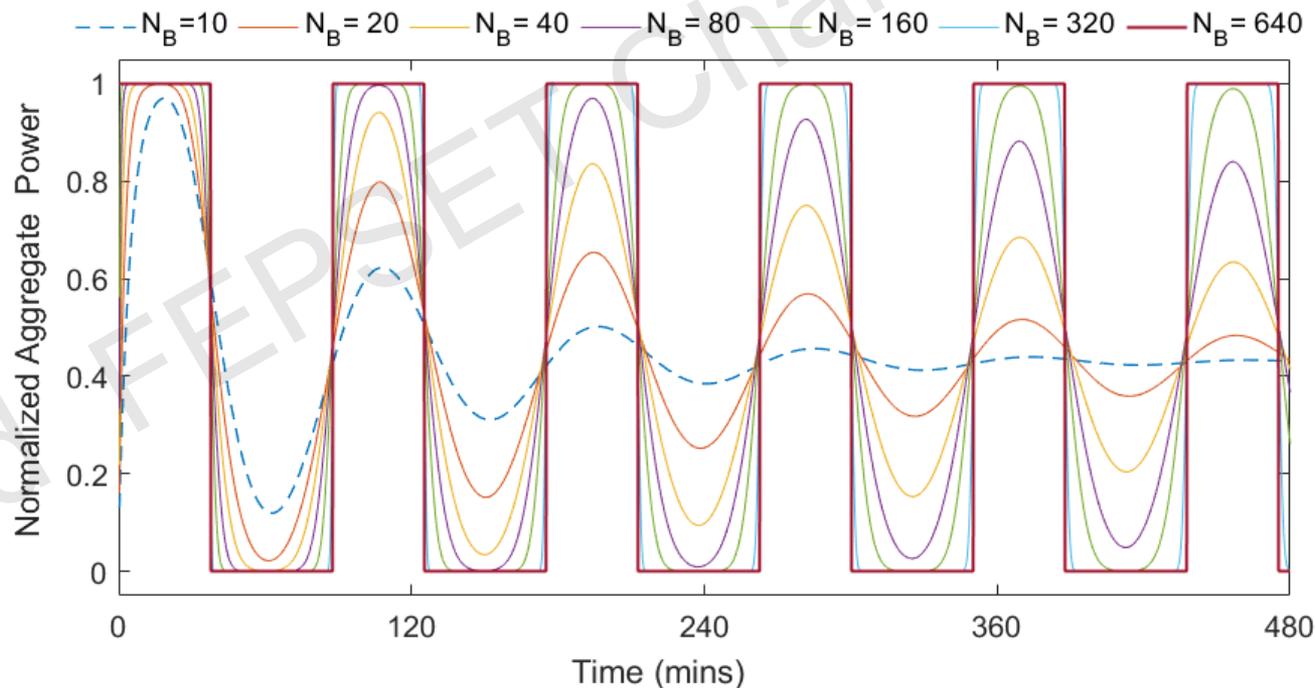
- Propagation of probability mass from one bin to another can be described by:

$$\dot{x} = Ax, \quad x_0 \text{ given}$$

where the state  $x(t)$  gives the probability mass in each bin at time  $t$ .

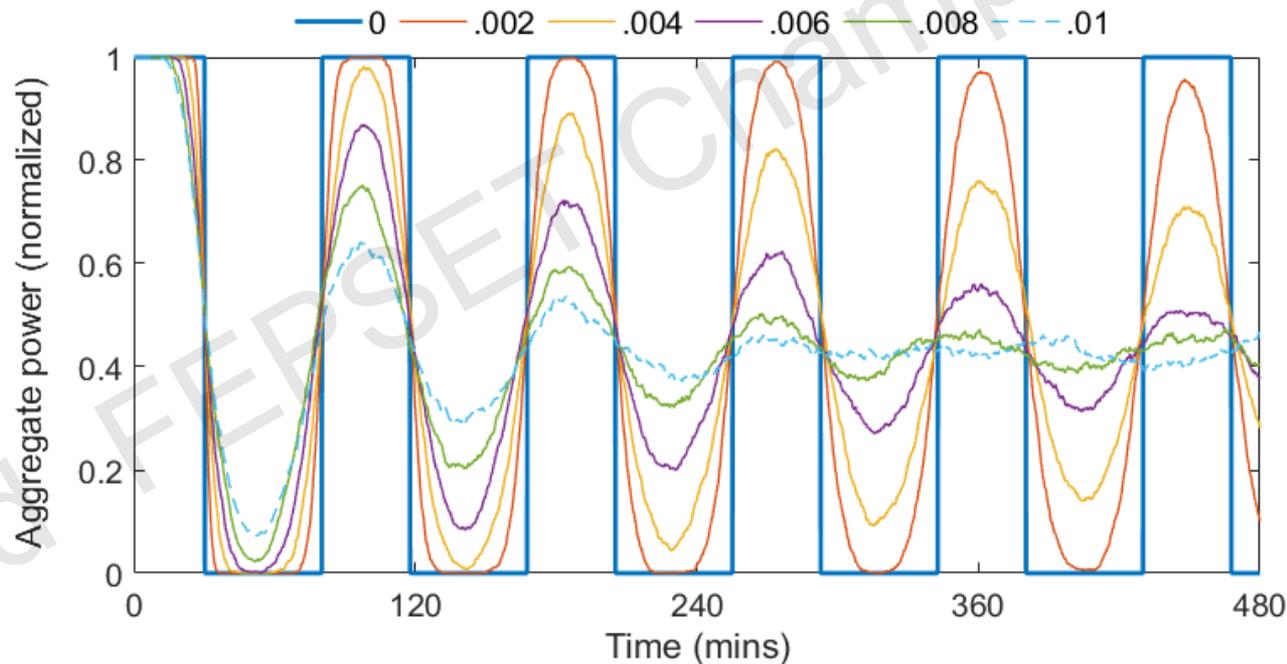
# Impact of bin width

- Consider a homogeneous population of TCLs with no noise.
- Assume an initial condition where all TCLs are in the same bin, having just switched on.
- Total power consumed by the ensemble, for different numbers of bins, displays quite different behaviour.

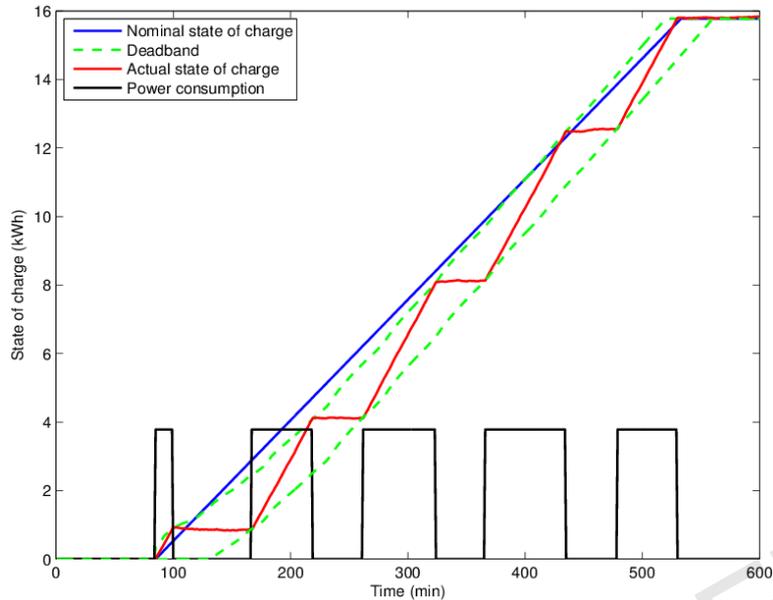


# Impact of background noise

- Homogeneous population but different levels of noise.
- Same initial condition as previously.
- Accurate model (high number of bins).

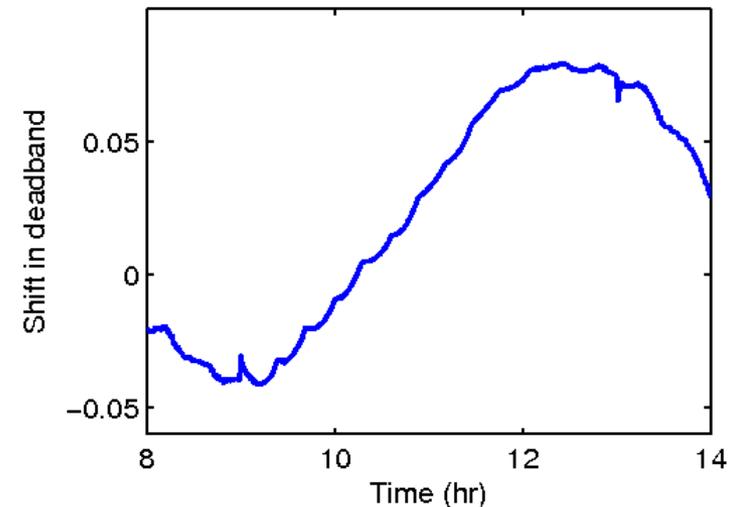
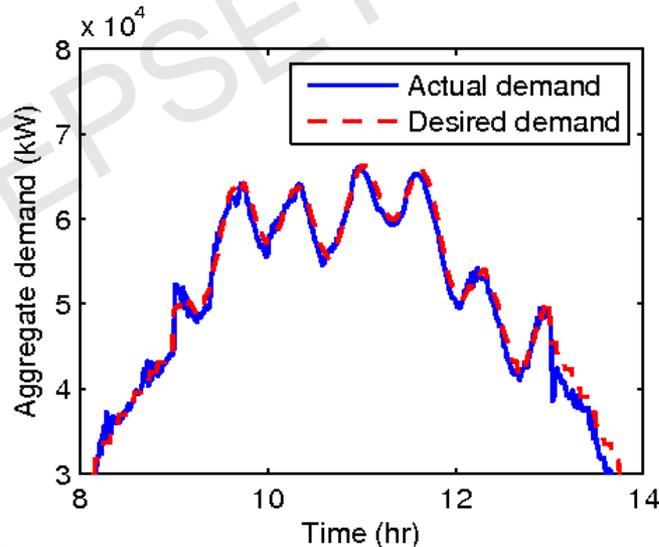


# Hysteresis-type control



- Hysteresis-based load control can be extended to loads that require a certain amount of energy, but have some flexibility in when they receive that energy.
  - PEV charging, refrigeration, dehumidifiers, pool pumps,...

## Tracking wind variability



# Strategies for controlling TCL ensembles

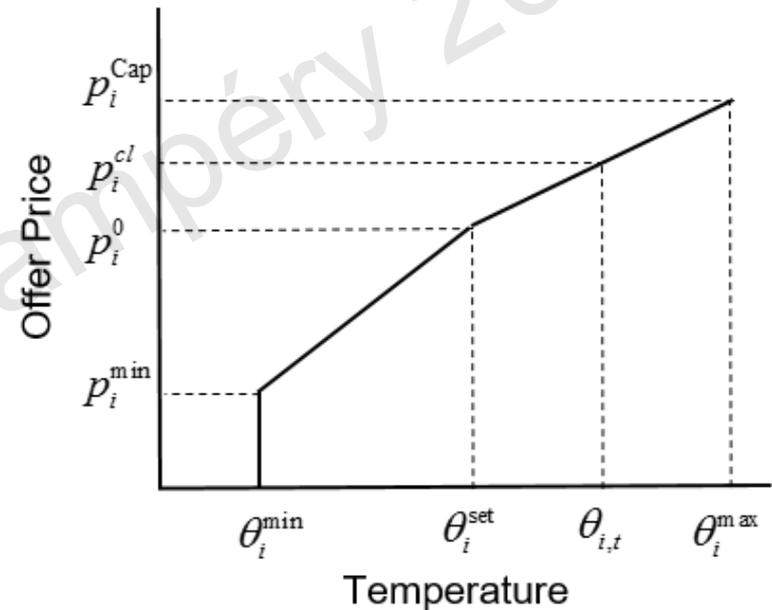
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- “Transactive” control.
- Variation of the set-point.
- There are many other possibilities (of course).
  
- Careful analysis is required to establish conditions under which behaviour is acceptable.



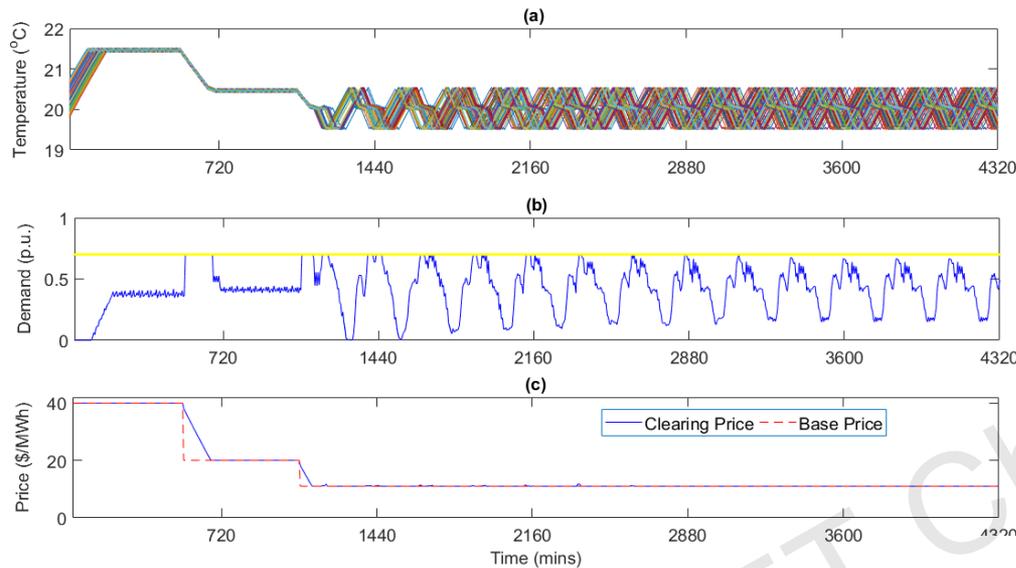
# “Transactive” control

- Based on a market mechanism, “prices to devices”.
- TCLs are equipped with “smart” thermostats that relate comfort to bidding price.
  - The bid is based on the temperature.
- Prices evolve according to the bin model dynamics  $\dot{x} = Ax$  over the period between market clearing times.
- The market clears periodically, e.g. every ten minutes.
  - All TCLs with a bid price above the clearing price  $\pi^{clr}$  are switched ON.
  - All TCLs with a bid price below the clearing price are switched OFF.
- This is described by a reset map:  $x^+ = B(\pi^{clr})x^-$



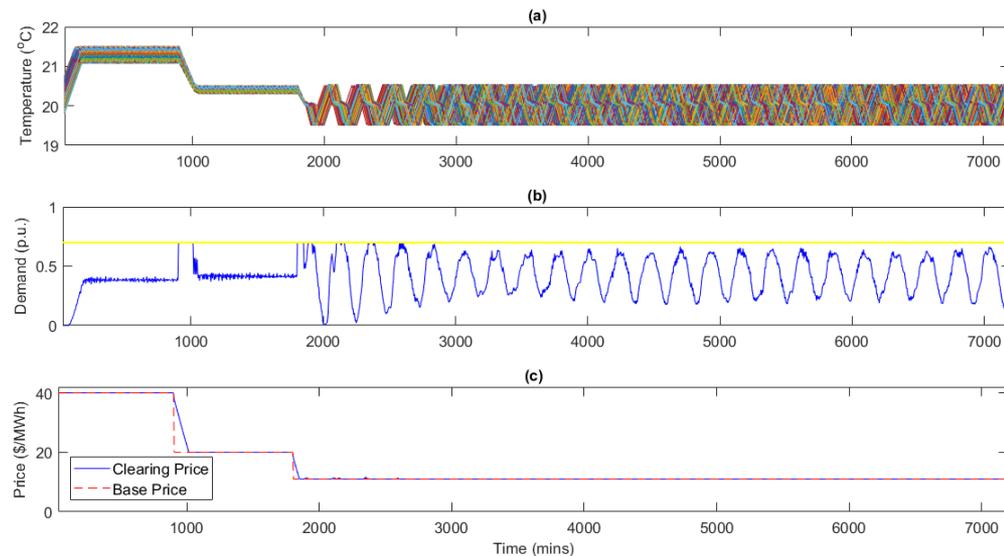
# TCL synchronization

Consider a distribution feeder with two large loads, e.g. EVs that are charging, together with numerous air-conditioners.



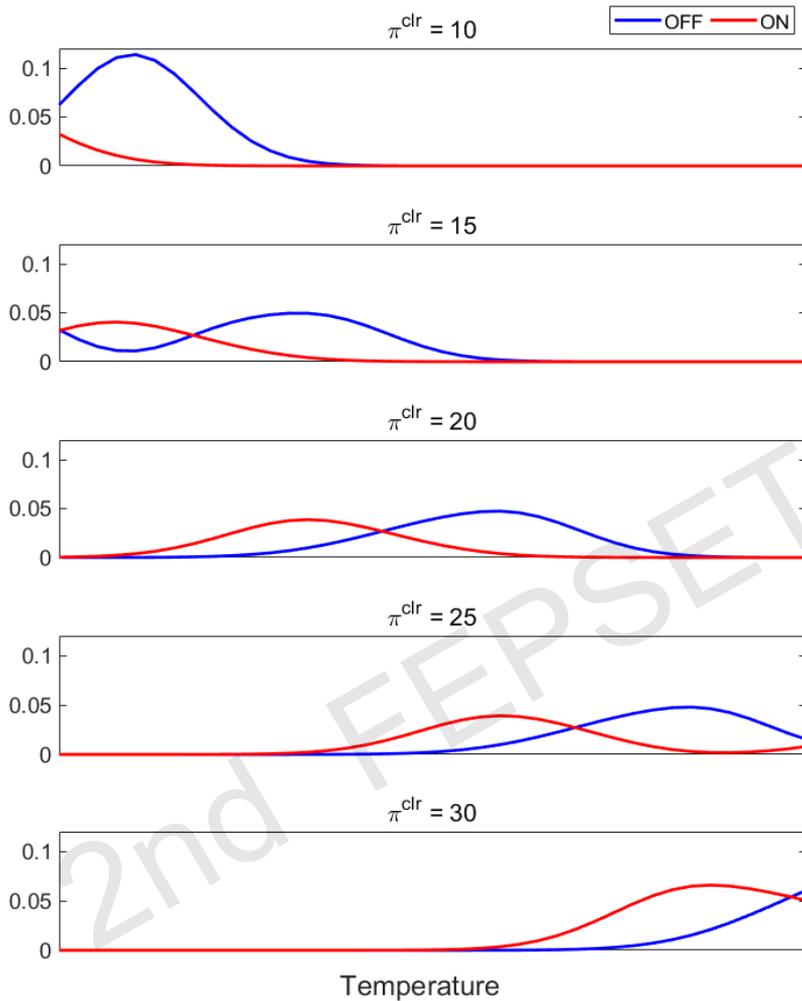
**Homogeneous population**

**Heterogeneous population**

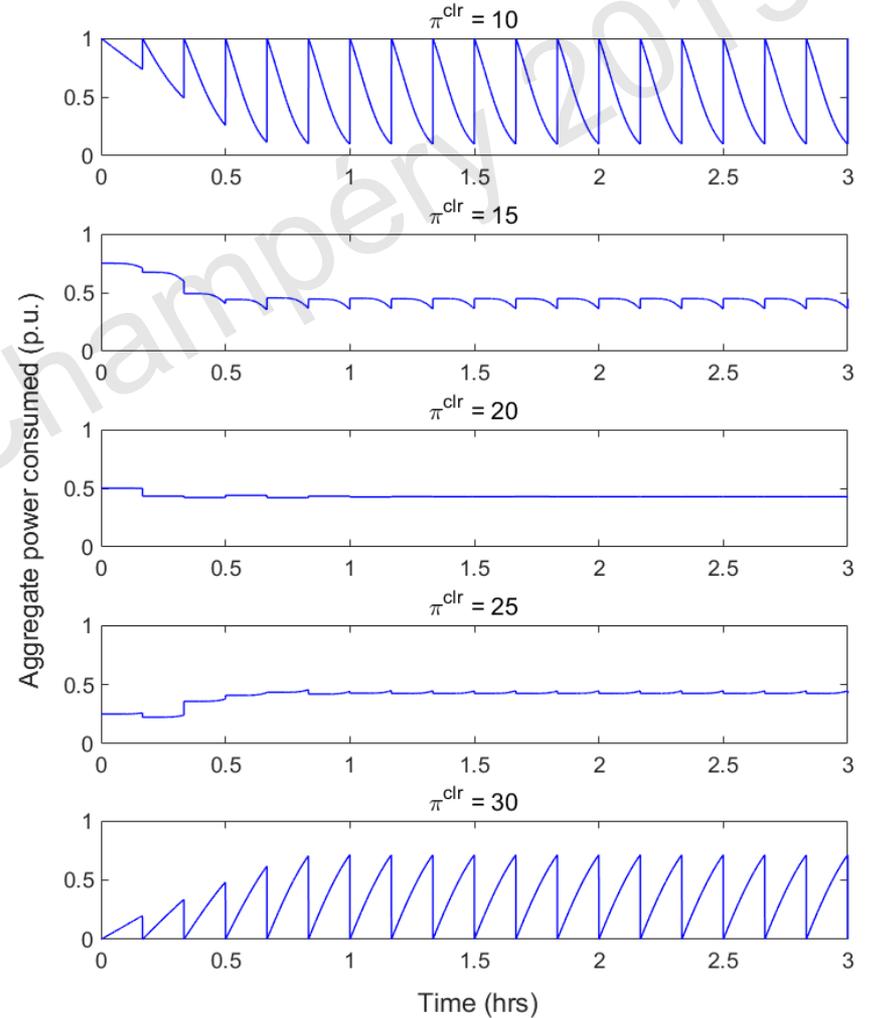


# Ensemble steady-state

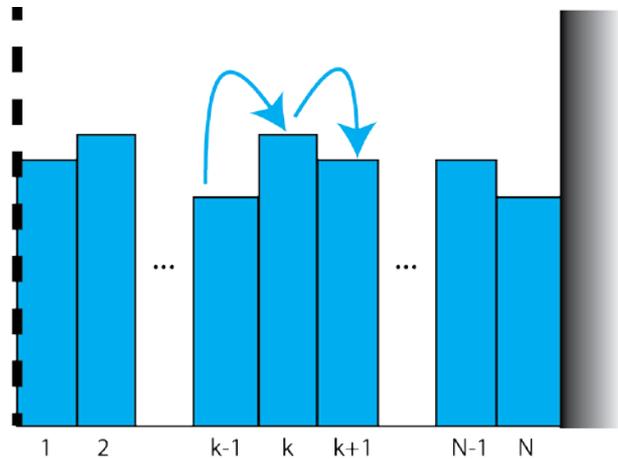
## ON/OFF distributions



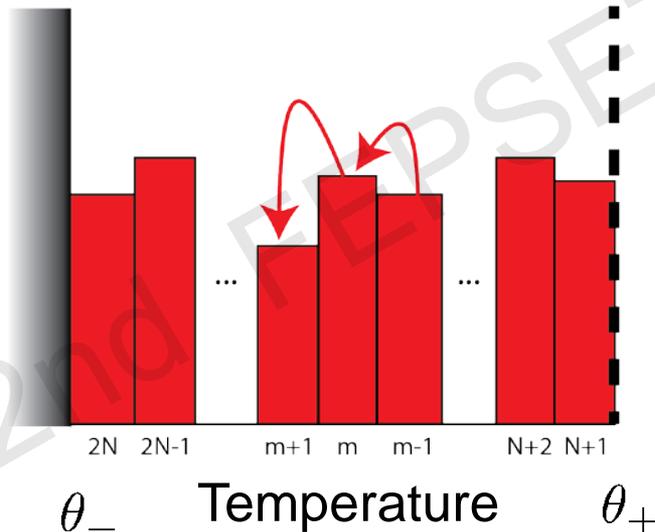
## Aggregate power consumption



# Set-point load control

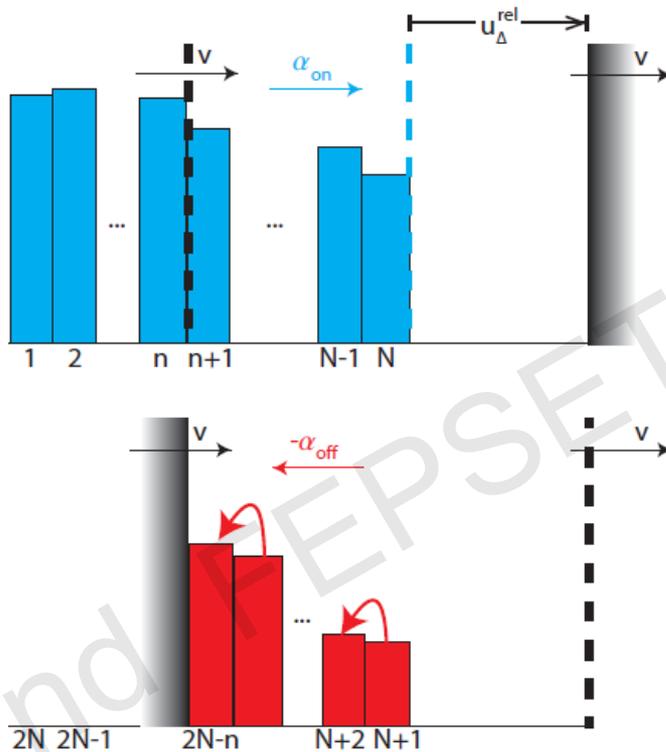


- Control strategy (for cooling loads):
  - Increase load by lowering set-point.
  - Decrease load by raising set-point.

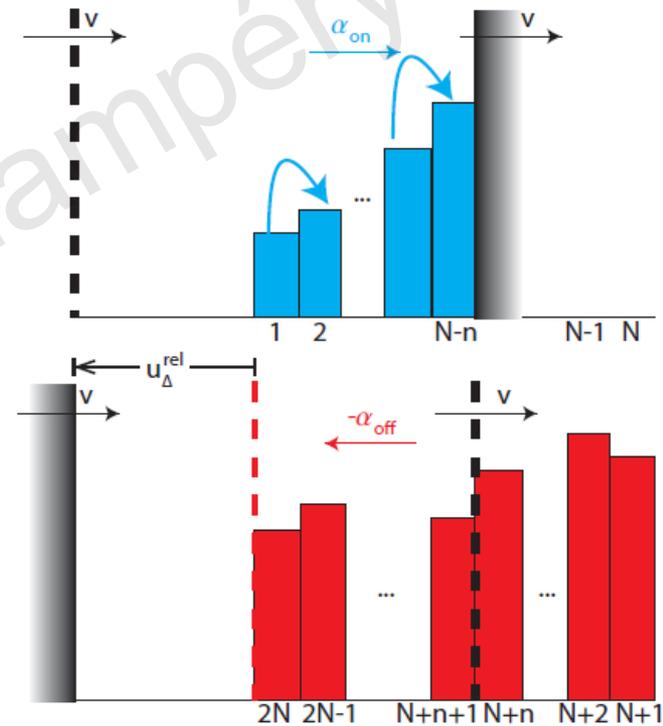


# Fast set-point changes

- If the set-point changes faster than the natural on/off rates:
  - Loads will migrate outside the hysteresis band.
  - State transitions cease to occur at one of the band limits.



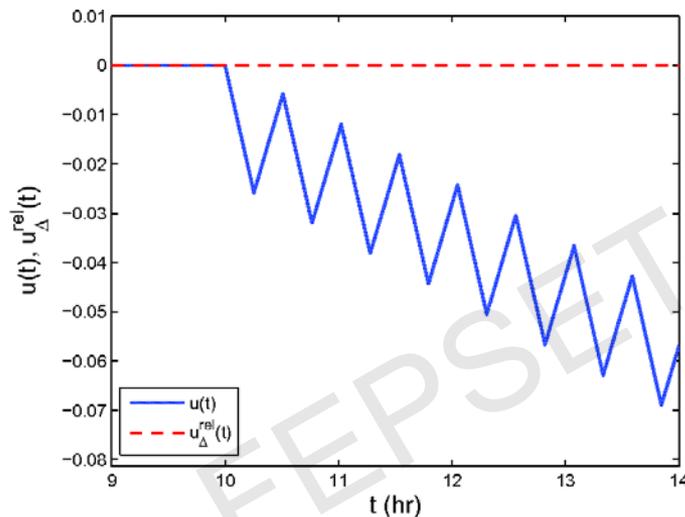
Fast increase in set-point  
(band moving to the right)



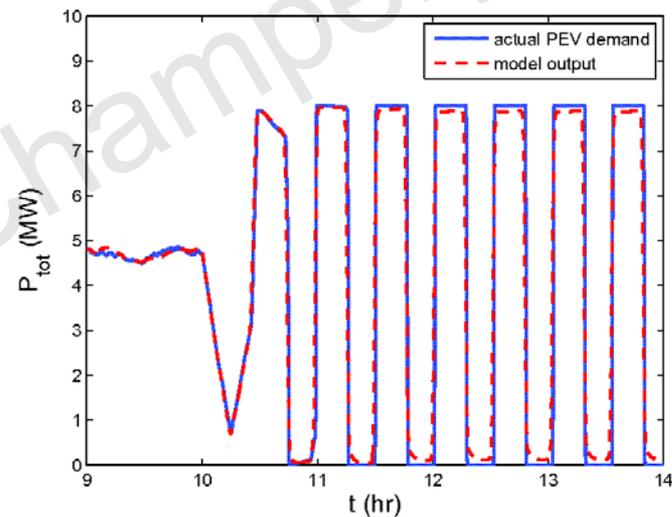
Fast decrease in set-point  
(band moving to the left)

# Nonlinear hybrid dynamics

- State-space modelling results in a nonlinear hybrid dynamical system.
  - Nonlinear because states and inputs multiply together.
  - Hybrid due to the influence of rapidly changing inputs.
- Example: 2000 EVs,  $P_{max} = 4$  kW for each EV.



Input (set-point change)



Output (total EV power demand)

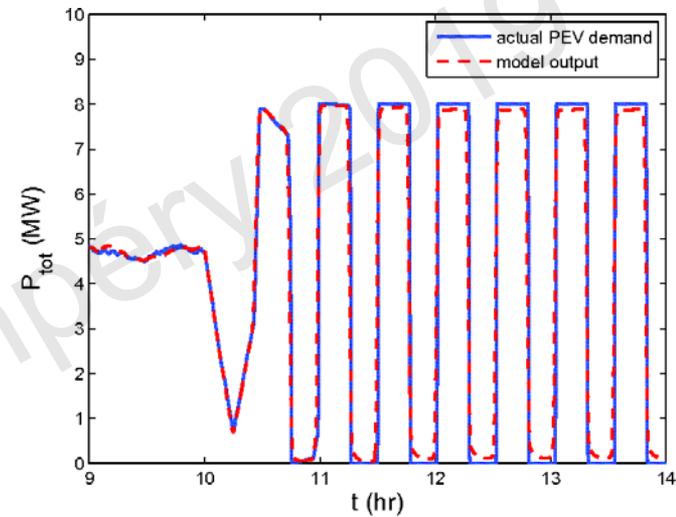
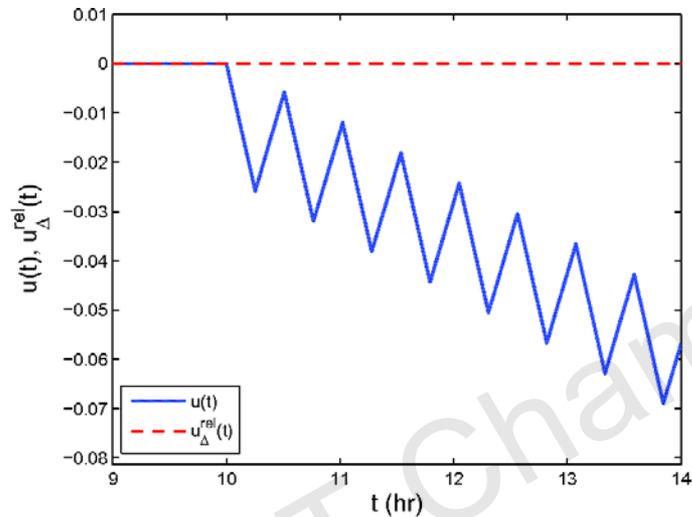
- In the output plot, the **red** curve (state-space model behaviour) and the **blue** curve (simulation of every EV) show very good agreement.



# Example: period-doubling bifurcation

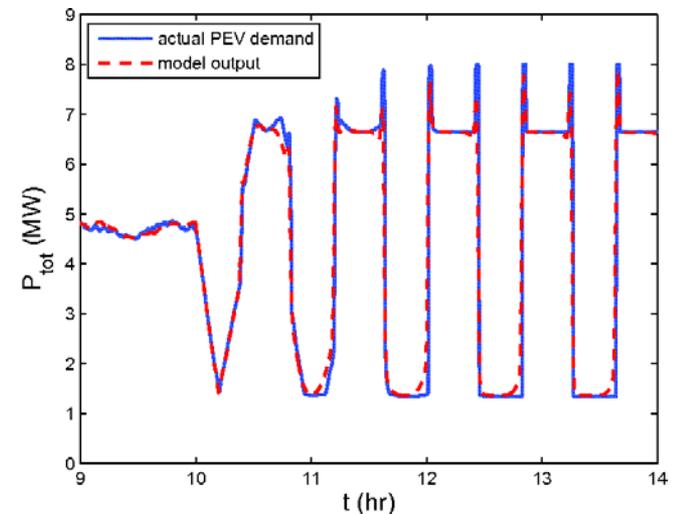
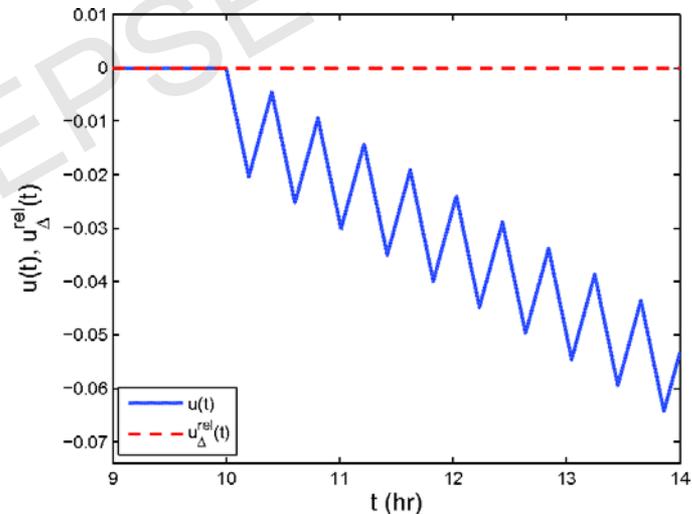
## Case 1:

- Period = 30.8 min
- Period-1 response



## Case 2:

- Period = 24.4 min
- Period-2 response



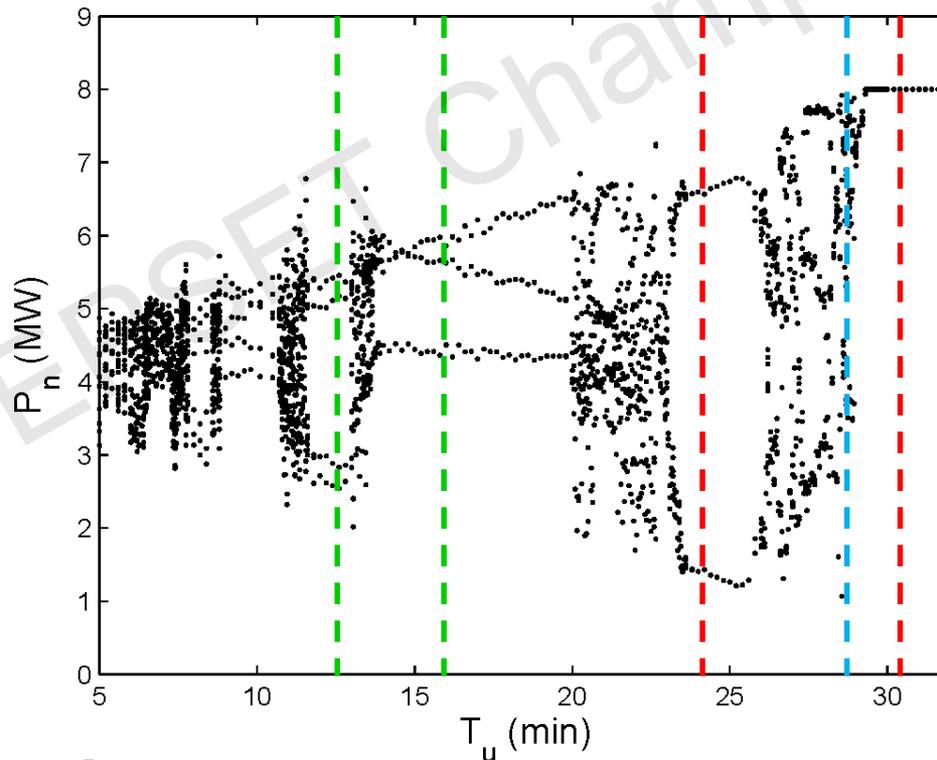
# Bifurcation diagram

- Analysis of period-adding bifurcations was achieved using the Poincare map:

$$P_n := P_{tot}(nT_u)$$

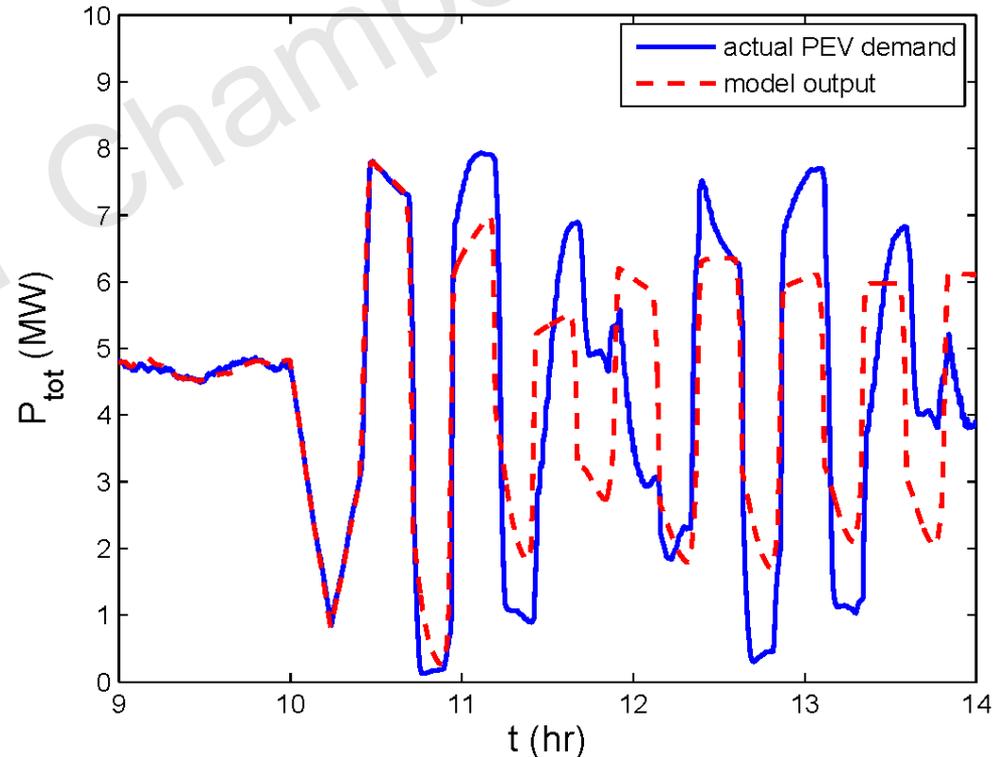
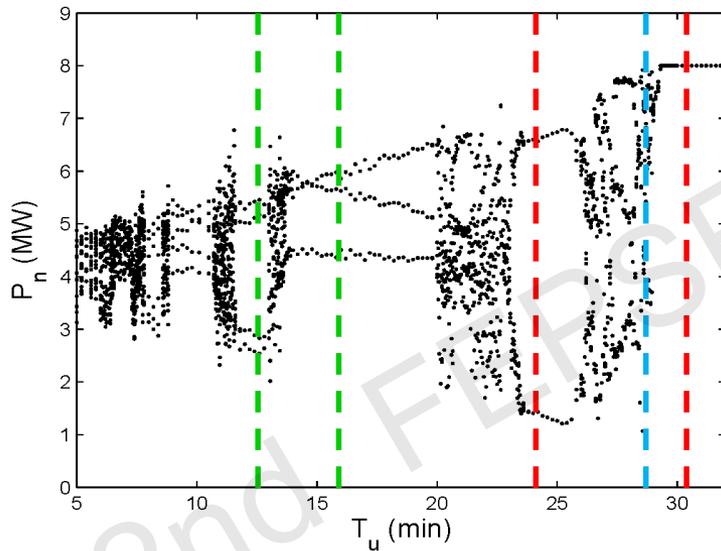
where  $T_u$  is the input period.

- Varying the input period  $T_u$  gave the bifurcation diagram:



# Chaos

- Periodic behaviour is separated by regions of aperiodic response.
- The accuracy of the state-space model reduces dramatically within the aperiodic regions.
  - This suggests high sensitivity and is indicative of chaos.



Input period = 28.8 min



# Conclusions

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- Significant actuation can be achieved through coordinated non-disruptive control of highly distributed loads.
- Hysteresis-based control of electrical loads may exhibit rich dynamical behaviour.
  - Structural stability of the system may be lost as crucial parameters are varied.
- A state-space model has been established to capture the response of hysteresis-based control of aggregations of loads.
  - The model is a hybrid dynamical system.
  - The model displays very good accuracy, except for parameter values that induce chaos-like behaviour.



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