Nonlinear Dynamics of Aggregate Load Models

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Motivation

- Growth in renewable generation makes balancing electricity production and consumption more difficult.

System-wide impact

The duck curve shows steep ramping needs and overgeneration risk

Local variability
Forms of non-disruptive load control

- Non-disruptive load control: energy users are oblivious to the control actions.

- Large individual loads.
  - Building HVAC control.

- Large numbers of small devices.
  - Thermostatically controlled loads (TCLs).
    - Air-conditioning, refrigeration, heat pumps.
    - Offer regulation capability and/or levelize renewable generation production.
  - Electric vehicle (EV) charging.
    - Prevent undesirable loading patterns.
    - Offer regulation capability for enhancing system operation.
Ensembles with natural dynamics

- The natural (hysteresis-based) dynamics of devices such as TCLs make regulation more challenging.
- A starting point is the development of a simplified model describing aggregate dynamic behaviour.

- The temperature associated with each TCL is influenced by random perturbations, e.g. opening doors/windows.
  - Modelled as noise.
- Every TCL has slightly different characteristics, e.g. thermal capacitance/resistance.
  - The population is heterogeneous.
Bin model approximation

- Regions (for cooling loads):
  - Blue loads are in the off state.
  - Red loads are in the on state.

- Propagation of probability mass from one bin to another can be described by:

\[ \dot{x} = Ax, \quad x_0 \text{ given} \]

where the state \( x(t) \) gives the probability mass in each bin at time \( t \).
Impact of bin width

- Consider a homogeneous population of TCLs with no noise.
- Assume an initial condition where all TCLs are in the same bin, having just switched on.
- Total power consumed by the ensemble, for different numbers of bins, displays quite different behaviour.
Impact of background noise

- Homogeneous population but different levels of noise.
- Same initial condition as previously.
- Accurate model (high number of bins).
Hysteresis-type control

- Hysteresis-based load control can be extended to loads that require a certain amount of energy, but have some flexibility in when they receive that energy.
  - PEV charging, refrigeration, dehumidifiers, pool pumps,...

Tracking wind variability
Strategies for controlling TCL ensembles

• “Transactive” control.
• Variation of the set-point.
• There are many other possibilities (of course).

• Careful analysis is required to establish conditions under which behaviour is acceptable.
“Transactive” control

- Based on a market mechanism, “prices to devices”.
- TCLs are equipped with “smart” thermostats that relate comfort to bidding price.
  - The bid is based on the temperature.
- Prices evolve according to the bin model dynamics $\dot{x} = Ax$ over the period between market clearing times.
- The market clears periodically, e.g. every ten minutes.
  - All TCLs with a bid price above the clearing price $\pi^{clr}$ are switched ON.
  - All TCLs with a bid price below the clearing price are switched OFF.
- This is described by a reset map: $x^+ = B(\pi^{clr})x^-$
TCL synchronization

Consider a distribution feeder with two large loads, e.g. EVs that are charging, together with numerous air-conditioners.

Heterogeneous population

Homogeneous population
Ensemble steady-state

**ON/OFF distributions**

- $\pi^{\text{clr}} = 10$
- $\pi^{\text{clr}} = 15$
- $\pi^{\text{clr}} = 20$
- $\pi^{\text{clr}} = 25$
- $\pi^{\text{clr}} = 30$

**Aggregate power consumption**

- $\pi^{\text{clr}} = 10$
- $\pi^{\text{clr}} = 15$
- $\pi^{\text{clr}} = 20$
- $\pi^{\text{clr}} = 25$
- $\pi^{\text{clr}} = 30$
Set-point load control

- Control strategy (for cooling loads):
  - Increase load by lowering set-point.
  - Decrease load by raising set-point.
Fast set-point changes

- If the set-point changes faster than the natural on/off rates:
  - Loads will migrate outside the hysteresis band.
  - State transitions cease to occur at one of the band limits.

**Fast increase in set-point**
(band moving to the right)

**Fast decrease in set-point**
(band moving to the left)
Nonlinear hybrid dynamics

- State-space modelling results in a nonlinear hybrid dynamical system.
  - Nonlinear because states and inputs multiply together.
  - Hybrid due to the influence of rapidly changing inputs.
- Example: 2000 EVs, $P_{max} = 4$ kW for each EV.

In the output plot, the red curve (state-space model behaviour) and the blue curve (simulation of every EV) show very good agreement.
Example: period-doubling bifurcation

Case 1:
- Period = 30.8 min
- Period-1 response

Case 2:
- Period = 24.4 min
- Period-2 response
Bifurcation diagram

- Analysis of period-adding bifurcations was achieved using the Poincare map:

\[ P_n := P_{tot}(nT_u) \]

where \( T_u \) is the input period.
- Varying the input period \( T_u \) gave the bifurcation diagram:
Chaos

- Periodic behaviour is separated by regions of aperiodic response.
- The accuracy of the state-space model reduces dramatically within the aperiodic regions.
  - This suggests high sensitivity and is indicative of chaos.

Input period = 28.8 min
Conclusions

• Significant actuation can be achieved through coordinated non-disruptive control of highly distributed loads.

• Hysteresis-based control of electrical loads may exhibit rich dynamical behaviour.
  – Structural stability of the system may be lost as crucial parameters are varied.

• A state-space model has been established to capture the response of hysteresis-based control of aggregations of loads.
  – The model is a hybrid dynamical system.
  – The model displays very good accuracy, except for parameter values that induce chaos-like behaviour.

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