Finite-Time Estimation of Time-Varying Frequency Signals in Low-Inertia Power Systems

J. G. Rueda-Escobedo\textsuperscript{1}, J. A. Moreno\textsuperscript{2}, J. Schiffer\textsuperscript{1}

\textsuperscript{1} Brandenburg University of Technology Cottbus-Senftenberg
\textsuperscript{2} Universidad Nacional Autónoma de México

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Motivation - Decreasing conventional actuation

Synchronous generator interfaced power plants – Operation & monitoring system

Electrical grid – Transmission Distribution – Frequency

Loads
Motivation - Decreasing conventional actuation

- **Synchronous generator interfaced power plants**
- **Operation & monitoring system**

**Electrical grid**

**Transmission Distribution**

**Solar homes & loads**

**More renewable energy production**

- Decreasing conventional actuation power
- Inverter-interfaced (renewable) units need to participate in frequency control
Most power inverters are operated in grid-feeding mode
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Frequency support possible via "virtual inertia control"

\[ P^{VI} = P_{\text{ref}} - K_P(\omega - \omega^d) - K_D\dot{\omega} \]

Inverter-interfaced generation: no rotor angular speed available

→ Local electrical frequency needs to be estimated from AC measurements at inverter terminals
Standard Phase-Locked Loop (PLL) scheme

- Most common frequency estimator: synchronous reference phase-looked loop (SRF-PLL) algorithm

\[ \dot{\hat{\theta}}(t) = \omega_0 + \Delta \hat{\omega}(t) \]

Phase detector (PD): \(abc\) to \(dq\) conversion of measured terminal voltage \(v_{abc}(t)\)

Loop filter (LF): tracking controller (usually PI)

Voltage-Controlled Oscillator (VCO): estimation of \(\hat{v}_q(\hat{\theta}(t))\) using

SRF-PLL is based on PI control \(\rightarrow\) only exact tracking of constant signals (same drawback applies to most other PLL variants)
PLLs can cause severe performance limitations.
Main contributions

- To provide a frequency estimator capable of exactly tracking a time-varying frequency signal of a symmetric three-phase AC waveform in finite time
- To demonstrate improved performance of proposed algorithm in estimating time-varying frequency signals compared to standard SRF-PLL via several numerical examples
- To show how this new frequency estimator can be used to provide "virtual inertia" support
Setup for frequency estimator design

• Symmetric three-phase AC signal with constant amplitude $A > 0$

$$v(t) = A \begin{bmatrix} \cos (\phi(t)) \\ \cos (\phi(t) - \frac{2}{3} \pi) \\ \cos (\phi(t) + \frac{2}{3} \pi) \end{bmatrix}$$

• $\phi : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ represents instantaneous phase and $\dot{\phi}(t) = \omega(t)$

• Symmetric AC signal in $\alpha\beta$ coordinates

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = T_{\alpha\beta} v(t) = A \begin{bmatrix} \cos (\phi(t)) \\ \sin (\phi(t)) \end{bmatrix}$$
Behavior of $y(t)$ over time is described by

$$\dot{y}(t) = \omega(t) \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} y(t) = \begin{bmatrix} -y_2(t) \\ y_1(t) \end{bmatrix} \omega(t)$$
Interpretation as parameter estimation problem

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- Classical problem of estimating constant parameters

\[
\dot{z}(t) = b^\top(t) \theta
\]

\( z : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^n \) is measured signal

\( b : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^{n \times m} \) is regressor

\( \theta \in \mathbb{R}^m \) is vector of unknown constant parameters
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- Main difference between both models:
  Frequency $\omega(t)$, i.e., unknown parameter of interest, may be time-varying
  → Classical estimation methods are not applicable
Time-varying parameter estimation problem

- Assumption: $\omega(t)$ is a Lipschitz function of time, i.e.,

$$|\omega(t_1) - \omega(t_2)| \leq \Delta^*|t_2 - t_1|, \forall t_1 \in \mathbb{R}_{\geq 0}, \forall t_2 \in \mathbb{R}_{\geq 0}, \Delta^* \in \mathbb{R}_{> 0}$$

- Then $\omega(t)$ is also absolutely continuous, which implies existence of a function $\xi : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$, such that (almost everywhere)

$$\dot{\omega}(t) = \xi(t), \quad |\xi(t)| \leq \Delta^* \leq \Delta$$

- Extended model:

$$\dot{y}(t) = b(t)\omega(t), \quad \dot{\omega}(t) = \xi(t), \quad b^\top(t) = [-y_2(t), y_1(t)]$$
Main result - I: Time-varying frequency estimator

- Proposed time-varying parameter estimator:

\[
\dot{\hat{y}}(t) = -k_1 \frac{\hat{y}(t) - y(t)}{\|\hat{y}(t) - y(t)\|^2} + b(t)\hat{\omega}(t),
\]

\[
\dot{\hat{\omega}}(t) = -k_2 b^\top(t) \frac{\hat{y}(t) - y(t)}{\|\hat{y}(t) - y(t)\|},
\]

- \(\hat{y}(t)\) and \(\hat{\omega}(t)\) represent estimates of \(y(t)\) and \(\omega(t)\)
- Constants \(k_1 > 0\) and \(k_2 > 0\) are tuning gains
- Nonlinear terms in algorithm

\[
\psi_1(\nu) := \frac{\nu}{\|\nu\|^\frac{1}{2}}, \quad \psi_2(\nu) := \frac{\nu}{\|\nu\|}, \quad \nu \in \mathbb{R}^2,
\]

which correspond to vector form of scalar functions \(|\cdot|^{\frac{1}{2}} \text{sign}(\cdot)|\) and \(\text{sign}(\cdot)\)
Main result - II: Sufficient convergence conditions

Theorem (Local finite-time convergence)

Let $c > 0$ and set estimator gains as

$$k_1 = \left(\frac{1}{4} + \sqrt{2}\right) A + c,$$

$$k_2 = \frac{9(5 + \sqrt{2})A}{8c} + \frac{9 + 40\sqrt{2}}{8} + \frac{5c}{2A} + \frac{\sqrt{2}\Delta}{c} + \frac{(1 + \sqrt{2)}\Delta^2}{\sqrt{2}Ac}$$

Then, if estimator is initialized such that

$$\frac{1}{8}Ah \geq \left(\frac{\lambda^+}{\lambda^-} \left(\|\hat{y}_0 - y_0\| + (\hat{\omega}_0 - \omega_0)^2\right)\right)^{\frac{1}{2}} \left(1 - \frac{1}{\eta^2}\right)$$

$\hat{\omega}(t)$ converges exactly to $\omega(t)$ in finite time

Parameters: $\lambda_{\pm} = \lambda_{\pm}(A, k_1, k_2)$ and $\eta = \eta(A, k_1, k_2)$

Main technical assumption: frequency $\omega(t)$ has an upper bound such that time period between sign changes of both $y_1(t)$ and $y_2(t)$ is lower bounded by some constant $h > 0$
Remarks on estimator design

- Proposed estimation algorithm combines time-varying version of super-twisting algorithm (STA) proposed in Guzmán and Moreno, IET Ctrl. Appl.’15, with its vector form in Nagesh and Edwards, Automatica’14

- For normalized signal with $A = 1$, estimate

$$\hat{y}(t) = \begin{bmatrix} \cos(\hat{\phi}(t)) \\ \sin(\hat{\phi}(t)) \end{bmatrix}$$

can be used in Park transformation

→ Proposed estimator provides same functionality as other PLL algorithms, but with better tracking performance in time-varying frequency scenario
Performance comparison with SRF-PLL - Ideal case

Frequency tracking exhibited by proposed estimator and SRF-PLL

Absolute value of frequency tracking error
Performance comparison with SRF-PLL - Noisy case

Frequency tracking exhibited by proposed estimator and SRF-PLL

Absolute value of frequency tracking error
Estimator application for virtual inertia control

\[
P_{\text{VI}} = -K_P (\hat{\omega} - \omega_d) - K_D \dot{\omega}
\]
Performance comparison with SRF-PLL - Virtual inertia

\[ \Delta P(t) = -e^{-0.1t} \sin(0.1t) \text{ [pu]} \]

STA ‘–’ SRF-PLL ‘- -’
Summary & future work

Main contributions

- Proposed time-varying version of super twisting algorithm to estimate instantaneous frequency of symmetric three phase AC signal
- In contrast to existing approaches, algorithm is capable of exactly tracking a time-varying frequency signal (if tuned according to proposed convergence conditions)
- Illustrated via simulations that improved virtual inertia performance can be achieved with proposed estimator compared to standard SRF-PLL

Future work

- Estimator improvement: global convergence result, broader set of admissible gains and extension of proposed algorithm to case of measurements corrupted by harmonics
- Virtual inertia control: provide tuning conditions for frequency estimator and controller, which ensure closed-loop stability; delay-robustness
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Energy-Based Control Design to Face the Challenges of Future Power Systems

Romeo Ortega
Laboratoire des Signaux et Systèmes
CNRS CENTRALE-SUPELEC
http://www.lss.supelec.fr/perso/ortega/index.html
ortega@lss.supelec.fr

Johannes Schiffer
Control Systems and Network Control Technology Group
Brandenburg University of Technology
http://www.b-tu.de/en/fg-regelungssysteme
schiffer@b-tu.de

Theoretical topics:
• Euler-Lagrange and port-Hamiltonian models
• Control by interconnection and PID-Passivity-based Control of nonlinear systems
• Adaptive control of nonlinear and nonlinearity parameterized systems

Practical examples:
• Power electronic systems: power converters and power factor compensation for nonlinear loads
• Control of alternative energy generating systems (wind power plants, fuel-cells and PV units)
• Power systems and microgrids: stabilization of low-inertia systems, distributed passivity-based control applications
• Electromechanical systems: sensorless control of motors, doubly-fed induction generators
• Energy management via control by interconnection